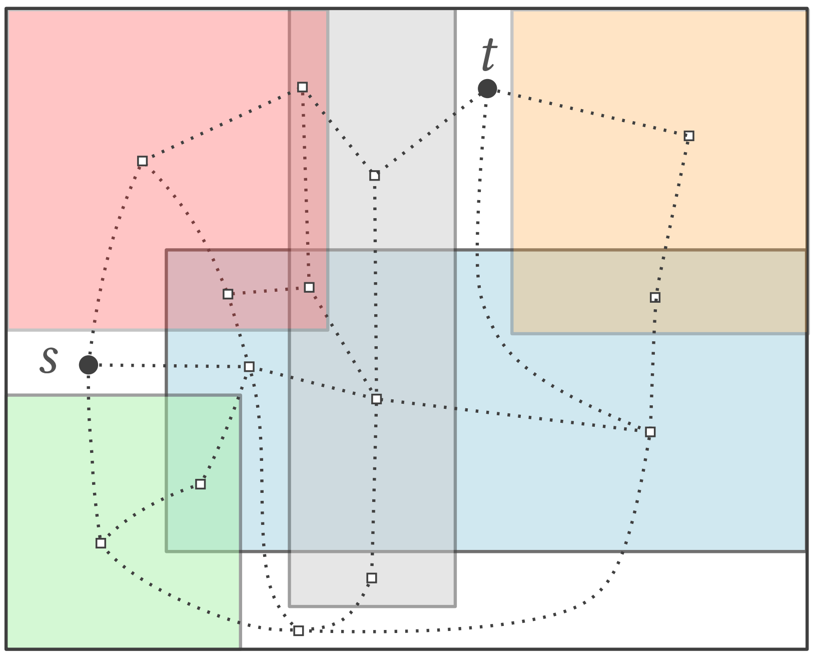
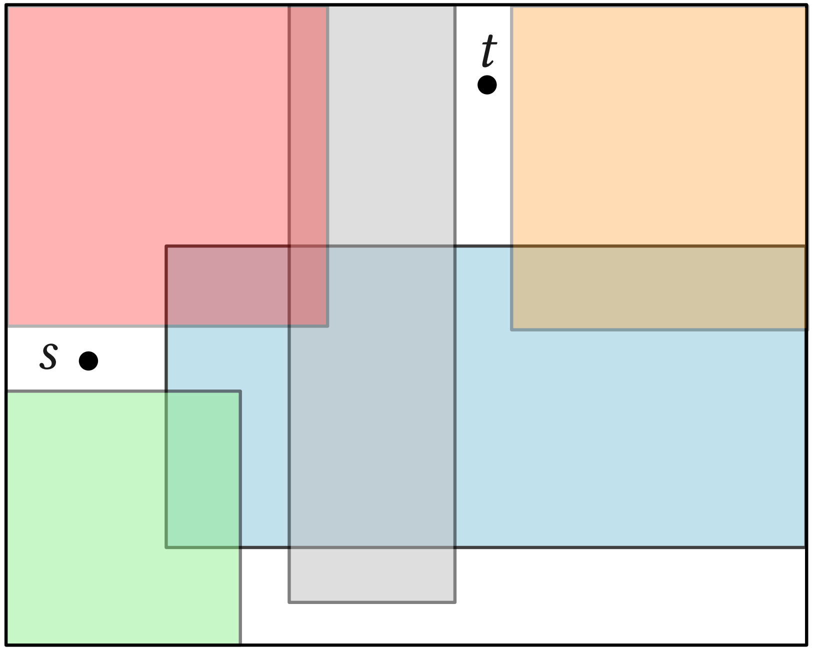


Improved Approximation Bounds for the Minimum Constraint Removal Problem

Sayan Bandyapadhyay, Neeraj Kumar, Subhash Suri and Kasturi Varadrajan

University of Iowa and UC Santa Barbara

Problem Description



Input : An arrangement of obstacles in plane, source s, target t

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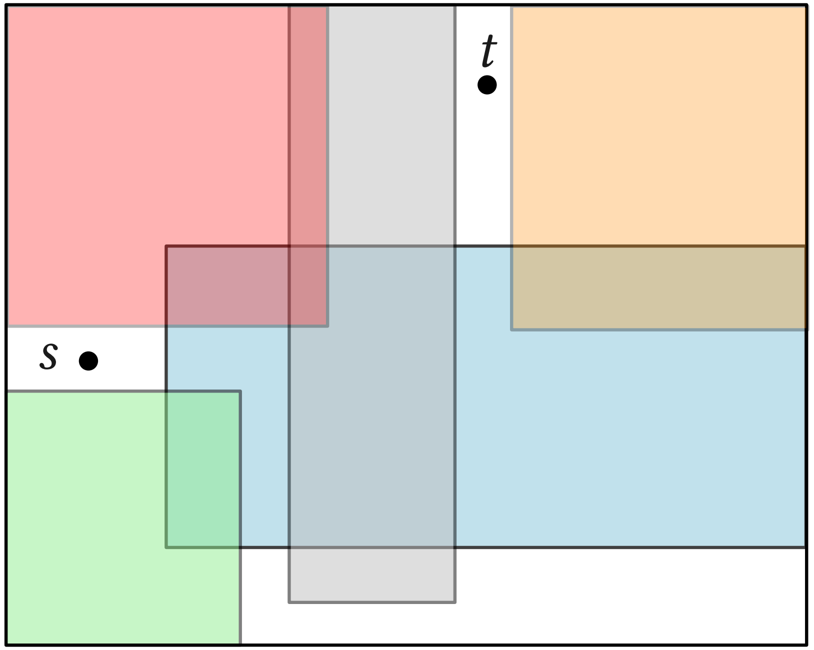

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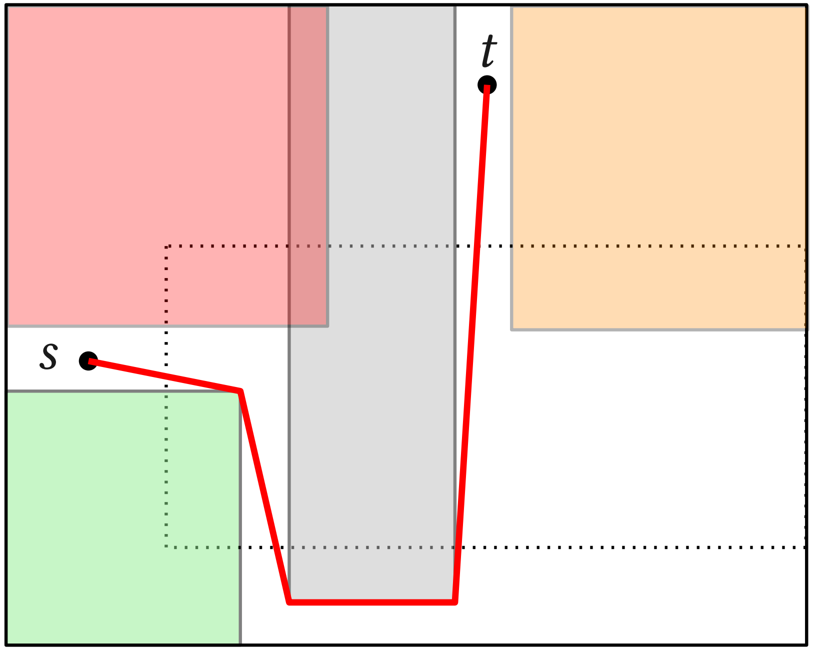

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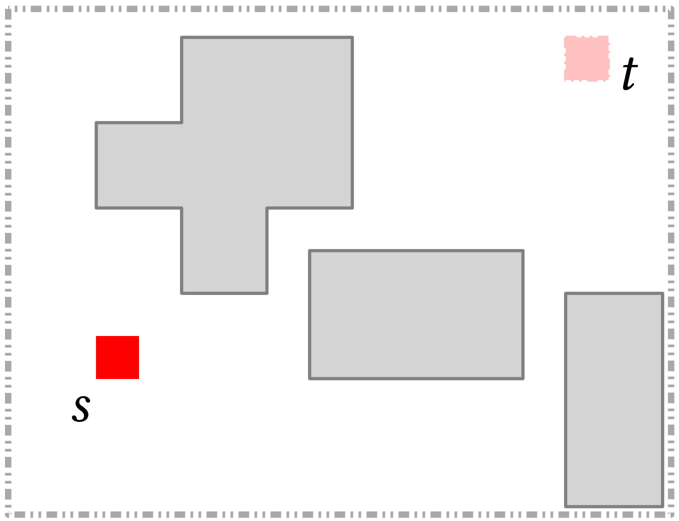
- **Robot Motion Planning**

  Configuration space approach:



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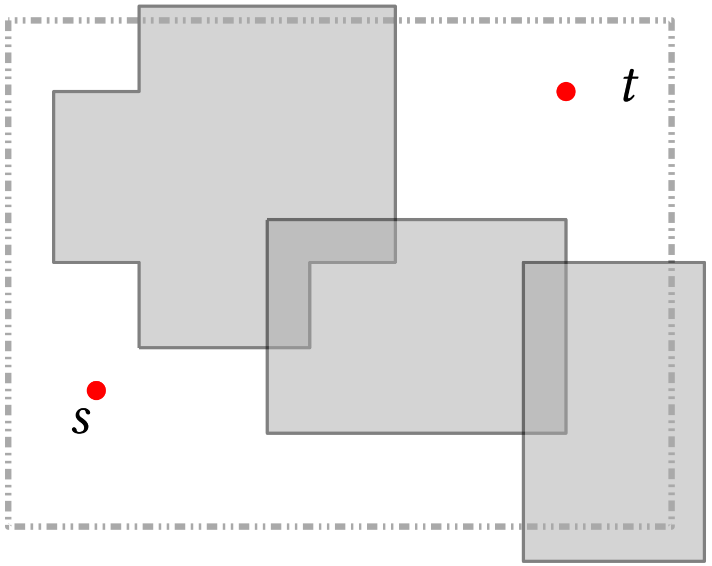

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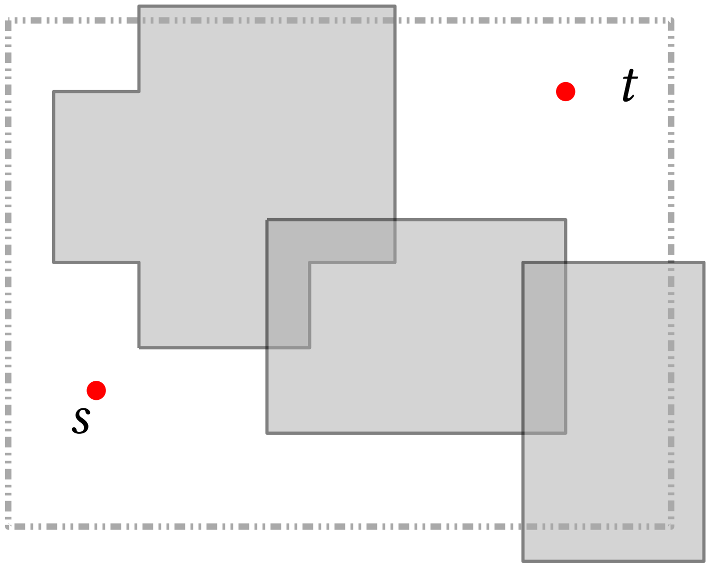

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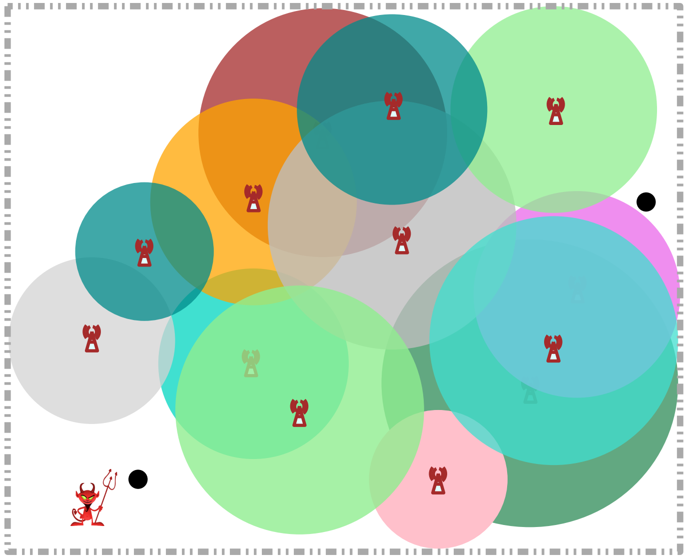

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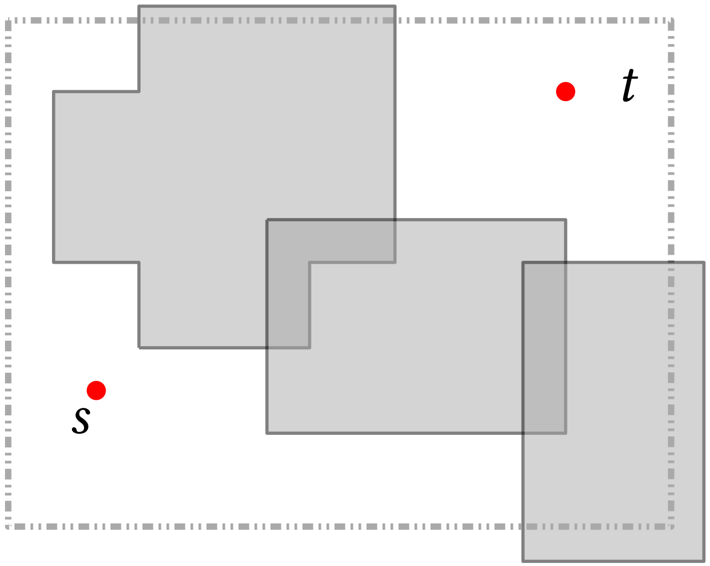

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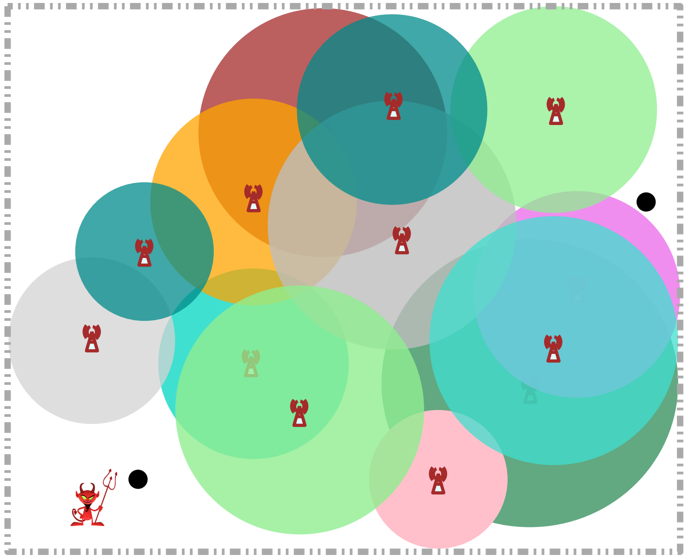

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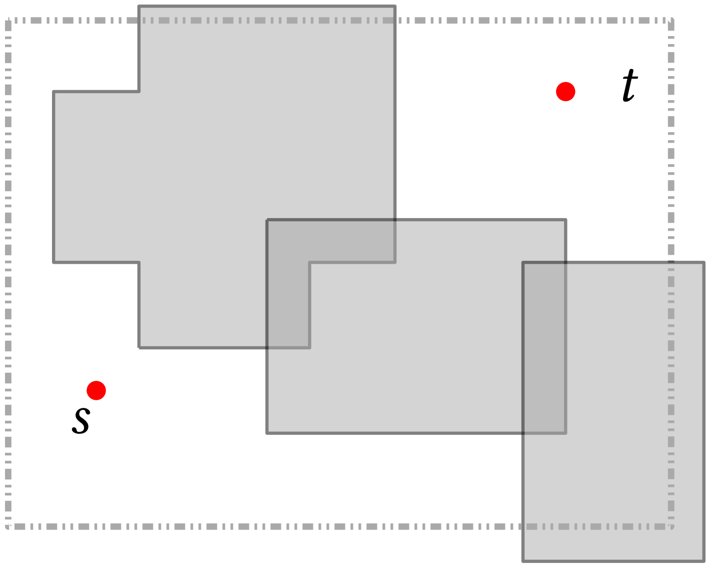

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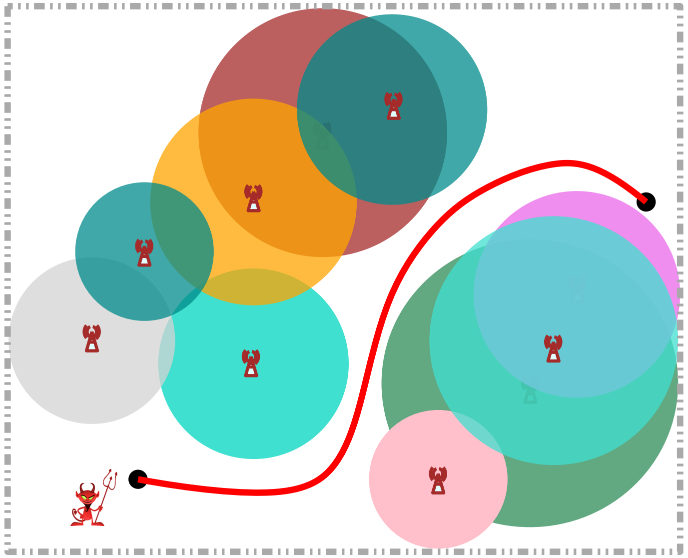

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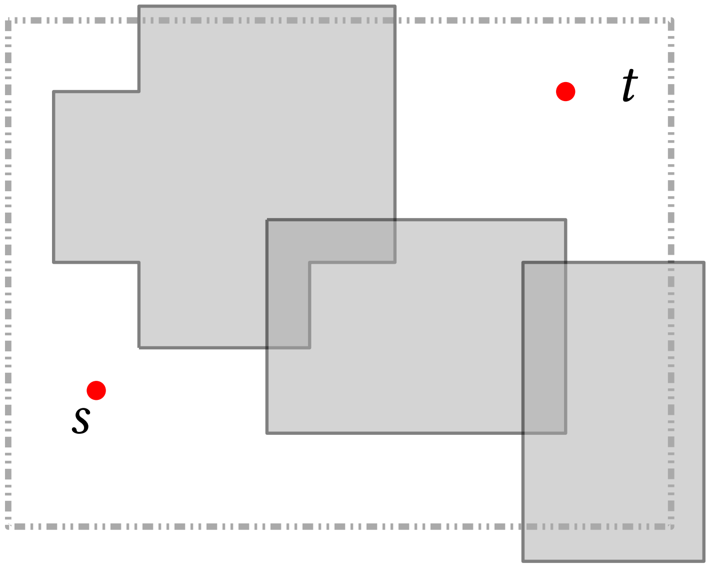
  [Polygonal Obstacles]



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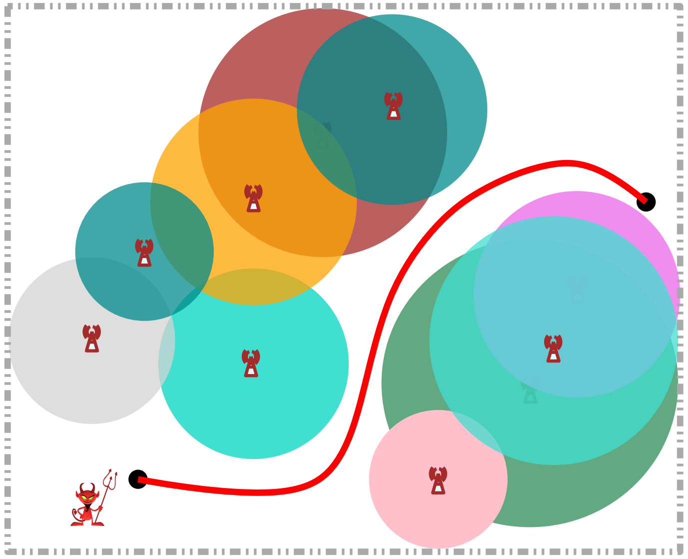
  Barrier Resillience ≡ Minimum number of sensors to disable

  [Disk Obstacles]



Previous Work

- Known to be intractable but no known approximations

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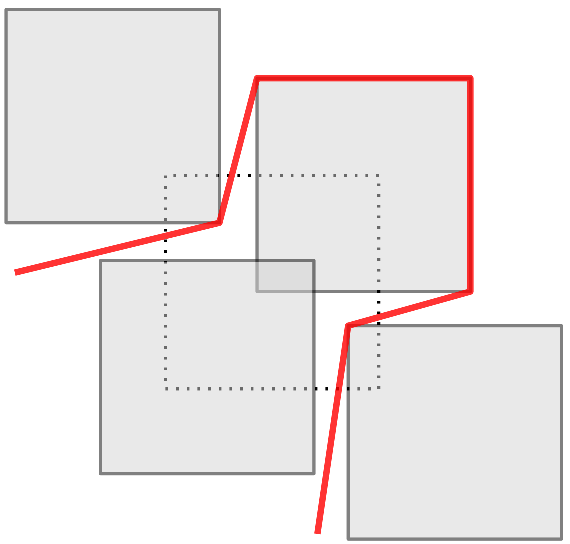
For unit square obstacles, this number is 2

– 2-approximation readily follows



Our Results

- An $O(\sqrt{n})$-approximation if input obstacles are rectilinear polygons
- An $O(\sqrt{n})$-approximation for arbitrary disk obstacles
- An $O(\sqrt{n}\alpha(n))$-approximation for arbitrary polygons

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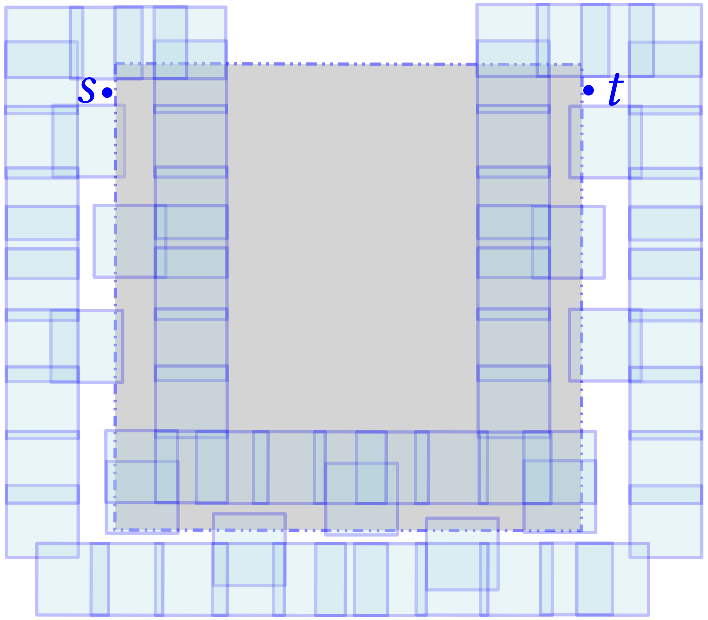
Rest of this talk

Main Idea

Previous work: Optimal path does not cross an obstacle a lot of times

For more general obstacles:



Even for axis-aligned squares

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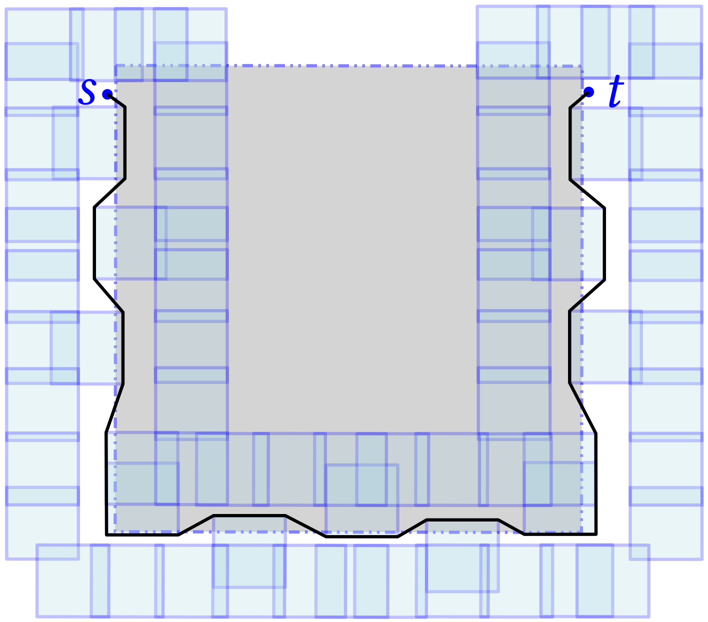
For more general obstacles:

– The optimal path can cross the same obstacle up to $\Theta(n)$ times



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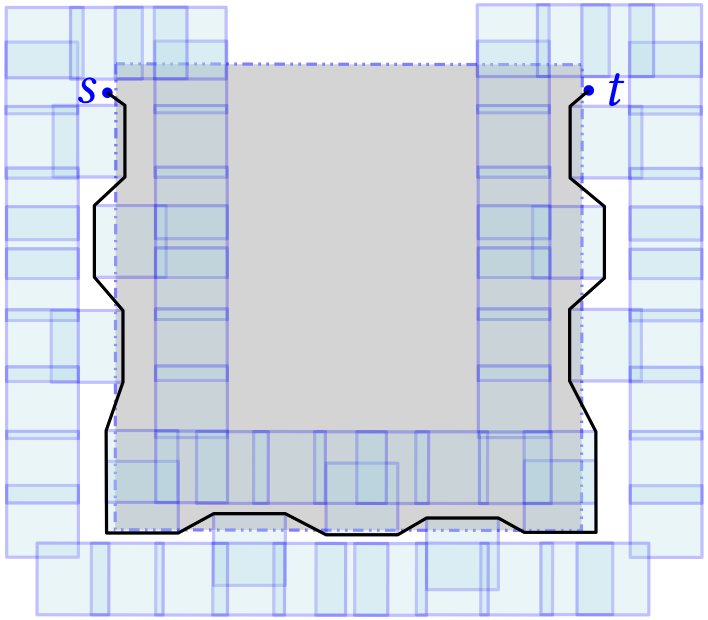

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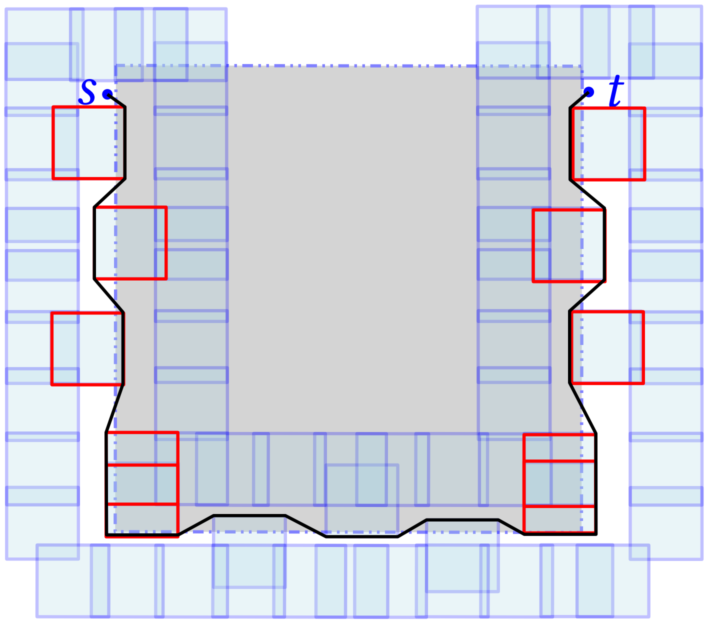


Even for axis-aligned squares

To alleviate this problem:

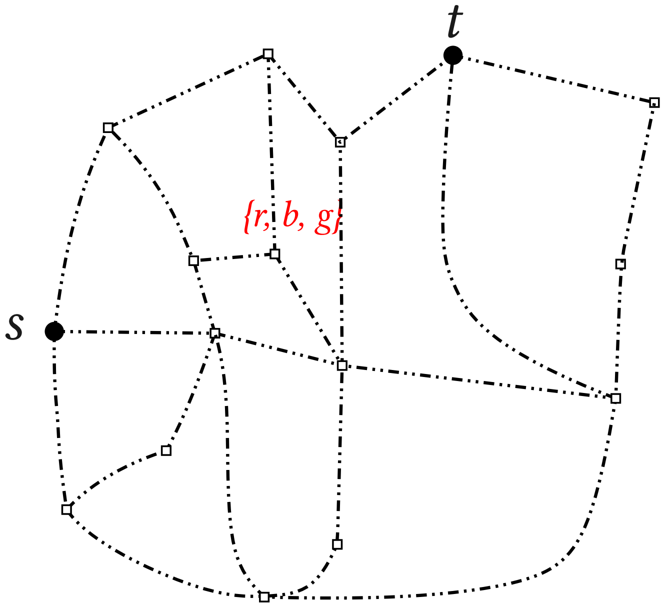
– we introduce a **filtering step**

Identify a “small set” of potentially dangerous obstacles

A Related Graph Problem

Input: Graph $G = (V, E)$, set of colors $\mathcal{C}$

Every vertex is assigned a subset $\mathcal{X}(v) \subseteq \mathcal{C}$



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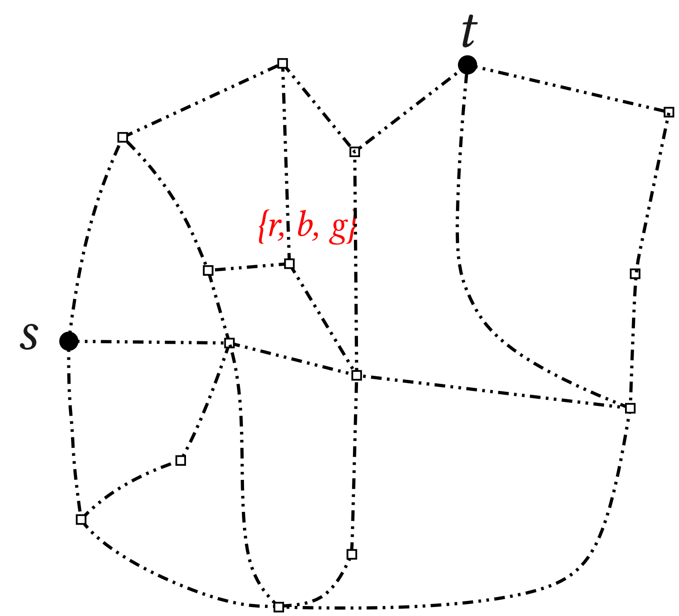

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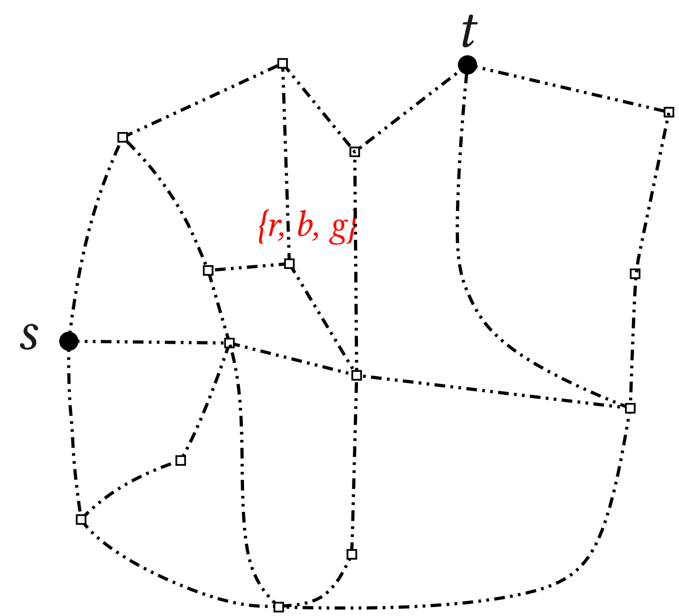

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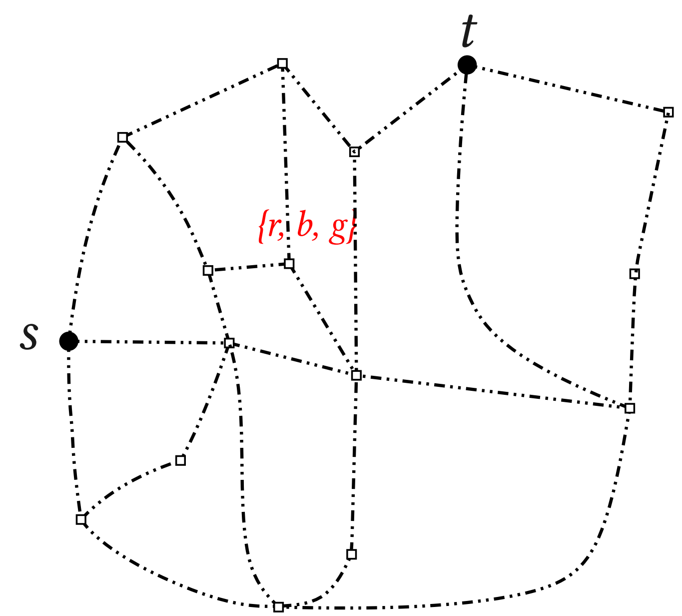

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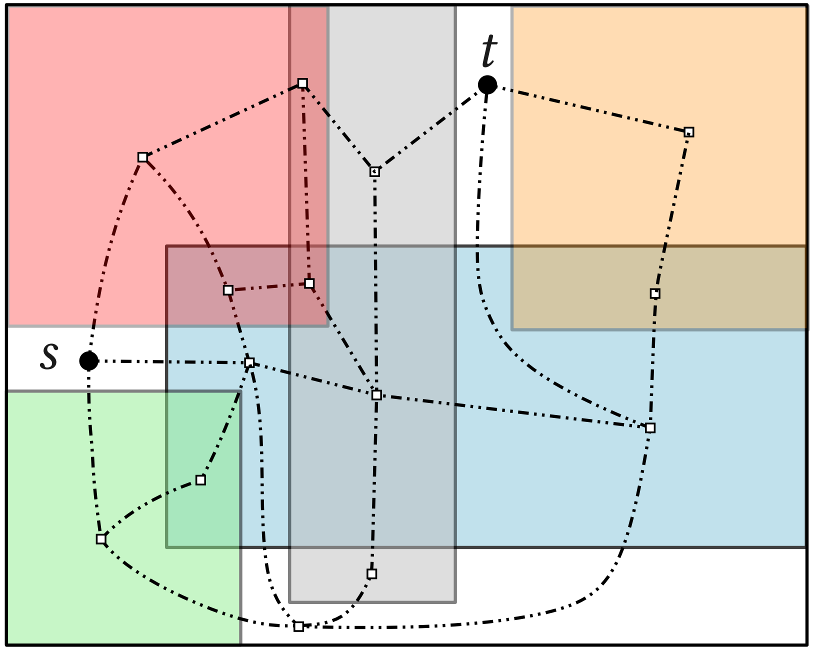

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Minimum Constraint removal can be cast as an instance of min-color path

An Approximation Framework

Input: A colored graph $G = (V, E, \mathcal{C})$, and an integer k

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Neighborhood $\mathcal{N} : C \rightarrow 2^{\mathcal{P}}$ is a mapping from colors to a subset of objects $\mathcal{P}$

can be any arbitrary set

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In other words, C appears on path "because of" an element in $\mathcal{N}(C)$

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$O(\sqrt{n})$-**approximation!**

$O(\sqrt{|V|})$-approximation for Min-Color Path

Suffices to just find the neighborhood $\mathcal{N}$

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$O(\sqrt{n})$**-approximation**

Does not really help with minimum constraint removal as $|V| = \Omega(n^2)$

With more effort, can still find a sparse neighborhood $\mathcal{N}$

Application to Geometric Objects

Step 1. Represent input as a colored graph $G = (V, E, \mathcal{C})$ such that:

- k-color path in G corresponds to path removing $\leq k$ obstacles
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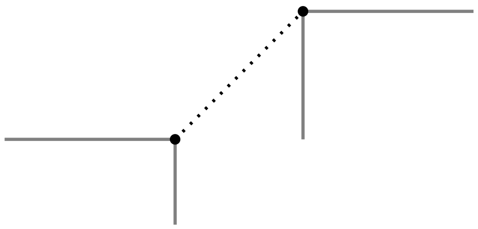
Step 2. Construct the neighborhood $\mathcal{N}$

Rectilinear Polygons

Graph Construction

$G = (V, E)$: complete graph over all n polygon vertices



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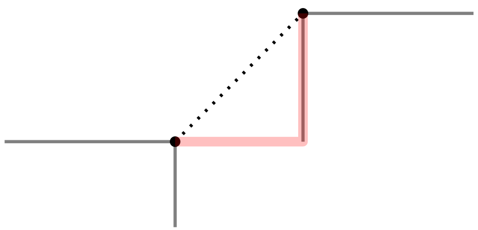

L-shaped embeddings of edges

To ensure k-color path in G corresponds to path removing $\leq k$ obstacles

– Fix embedding e_{uv} of an edge (u, v)

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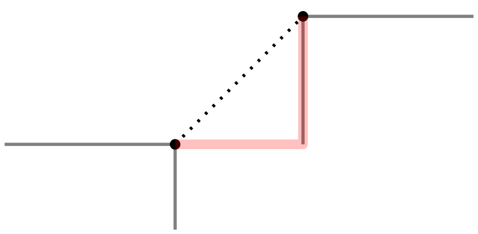

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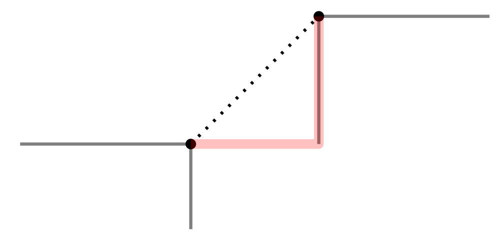

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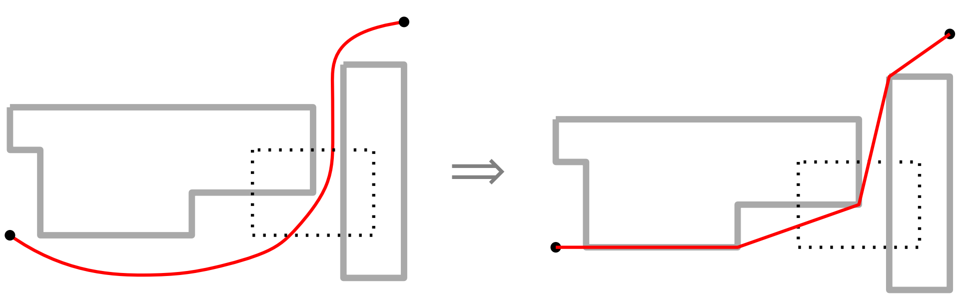
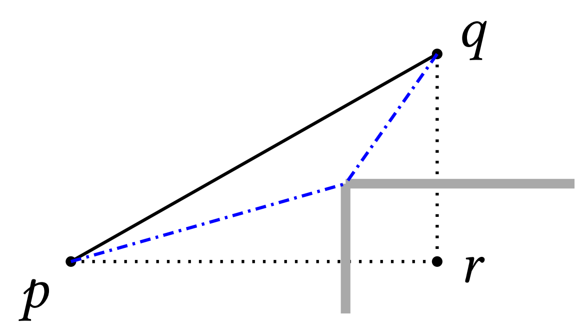


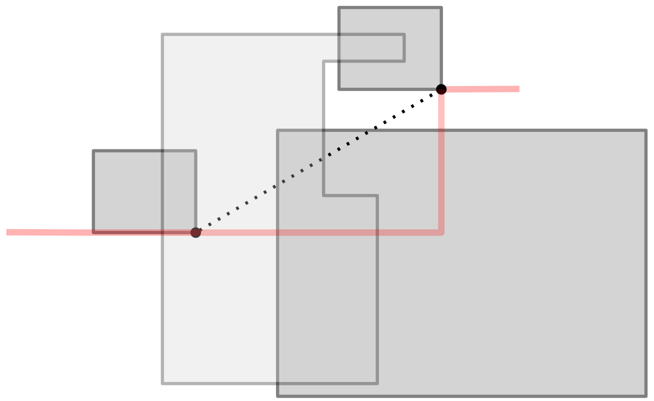


Rectilinear Polygons

Neighborhood Construction

What causes the path to cross a given obstacle?

–An obstacle corner



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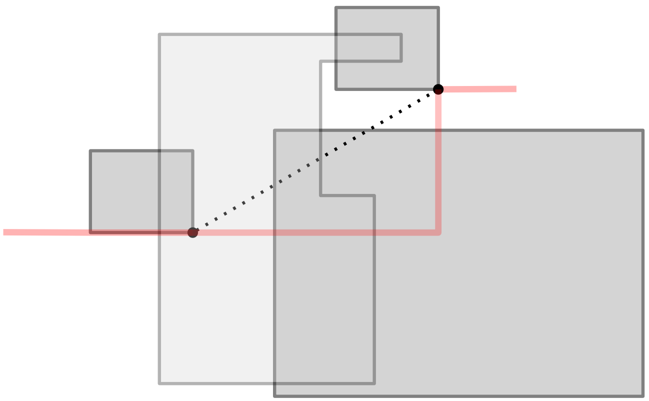

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Obvious candidate for the set $\mathcal{P}$: set of all corners

How to add corners to $\mathcal{N}(C)$ of obstacle C ensuring small total size?

Rectilinear Polygons

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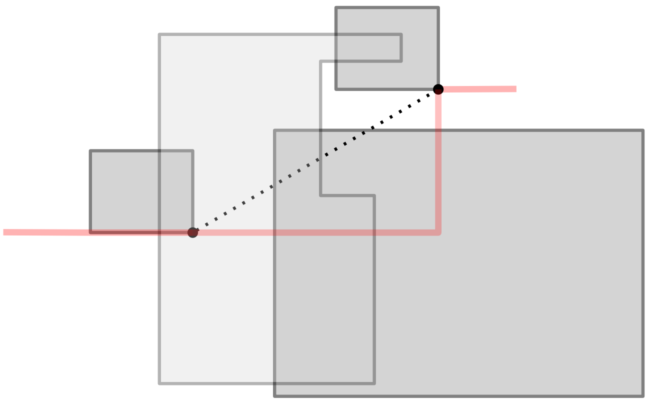

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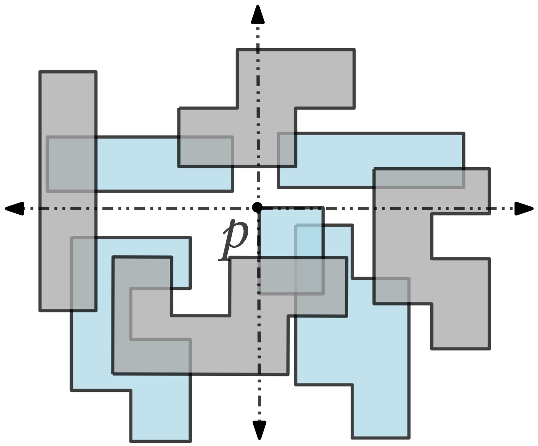
Obvious candidate for the set $\mathcal{P}$: set of all corners

How to add corners to $\mathcal{N}(C)$ of obstacle C ensuring small total size?



– Shoot a ray at a corners p

– Include p to neighborhood of first k obstacles

Rectilinear Polygons

Neighborhood Construction

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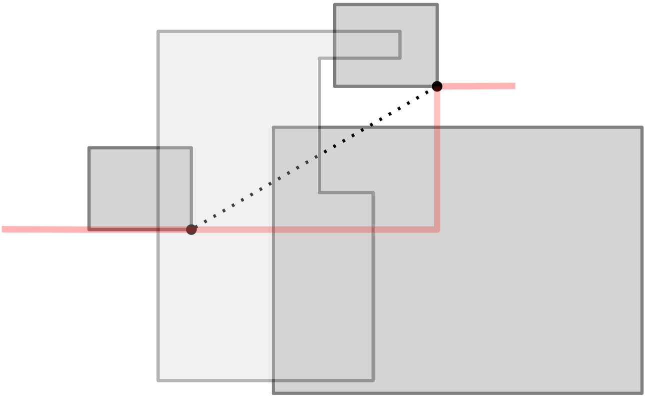

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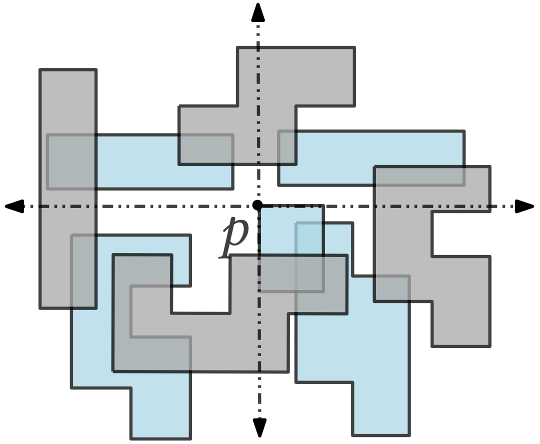

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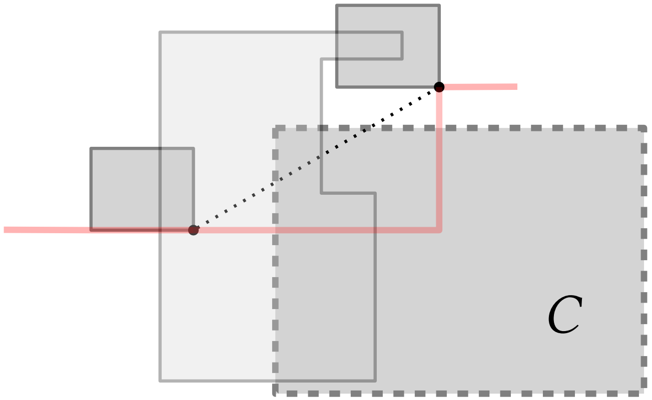
Total neighborhood size = $4k \cdot n$

Rectilinear Polygons

Each crossing of C by a valid k-color path is charged to a *neighbor* corner

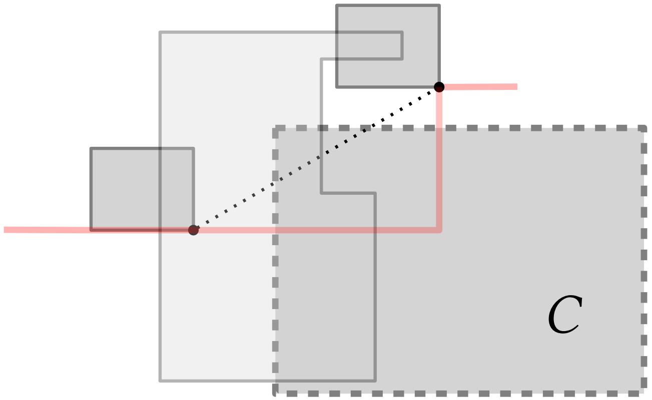


Rectilinear Polygons

Each crossing of C by a valid k-color path is charged to a *neighbor* corner

Each corner is charged for C at most twice

C is crossed $O(|\mathcal{N}(C)|)$ times



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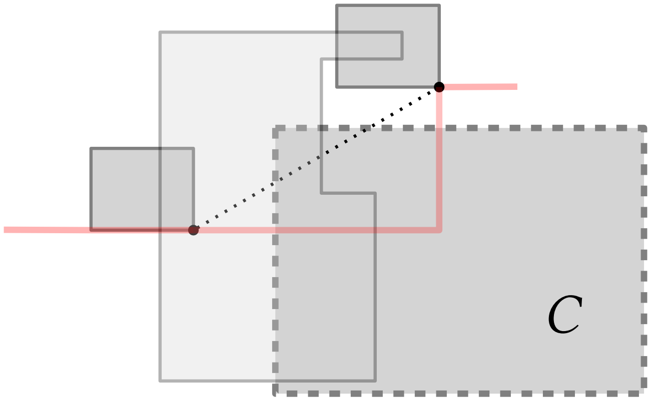

Both bounded size and bounded occurrence propertiey are satisfied

$O(\sqrt{n})$**-approximation!**

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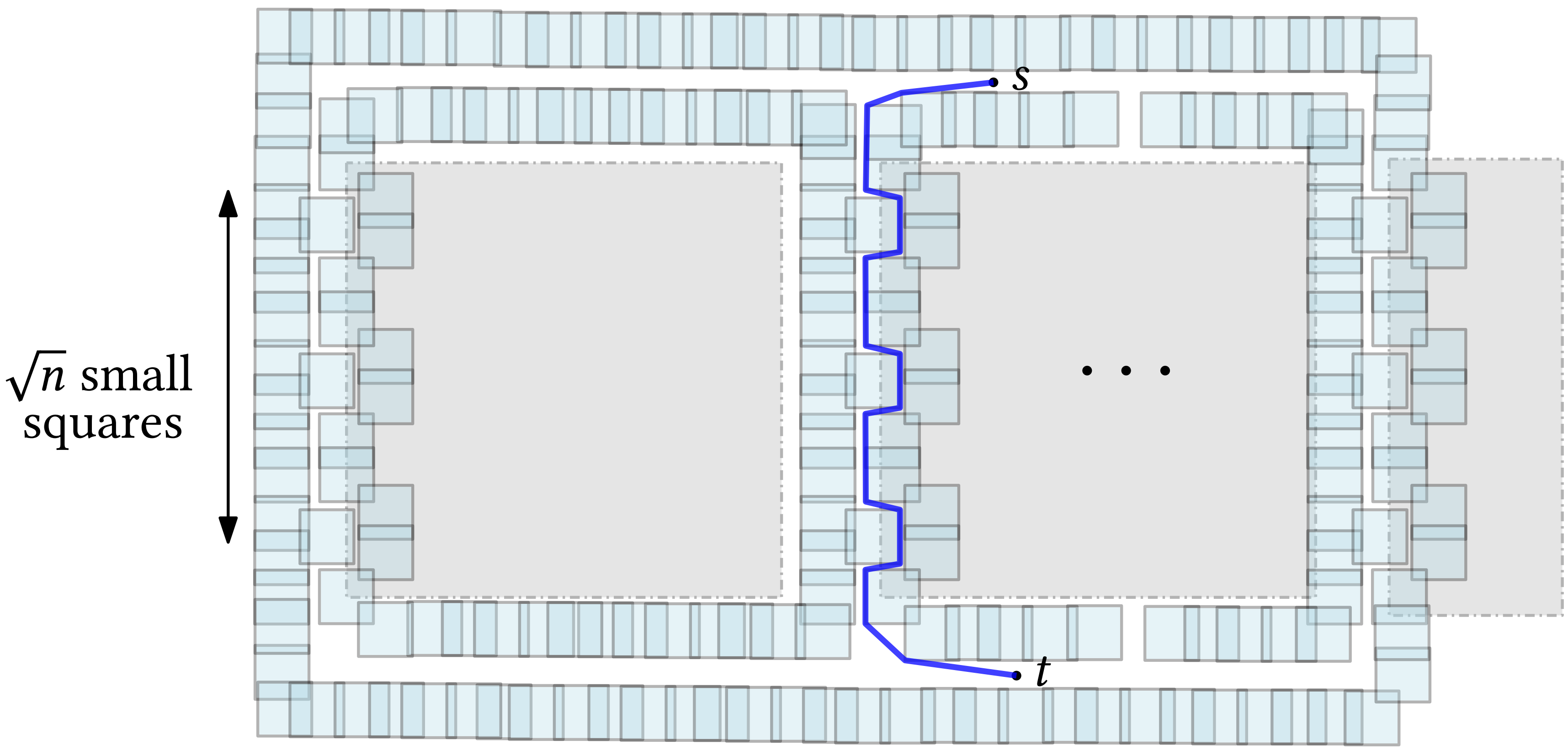
Thanks!

Backup : Tight Example



Create $\Theta(n)$ such channels