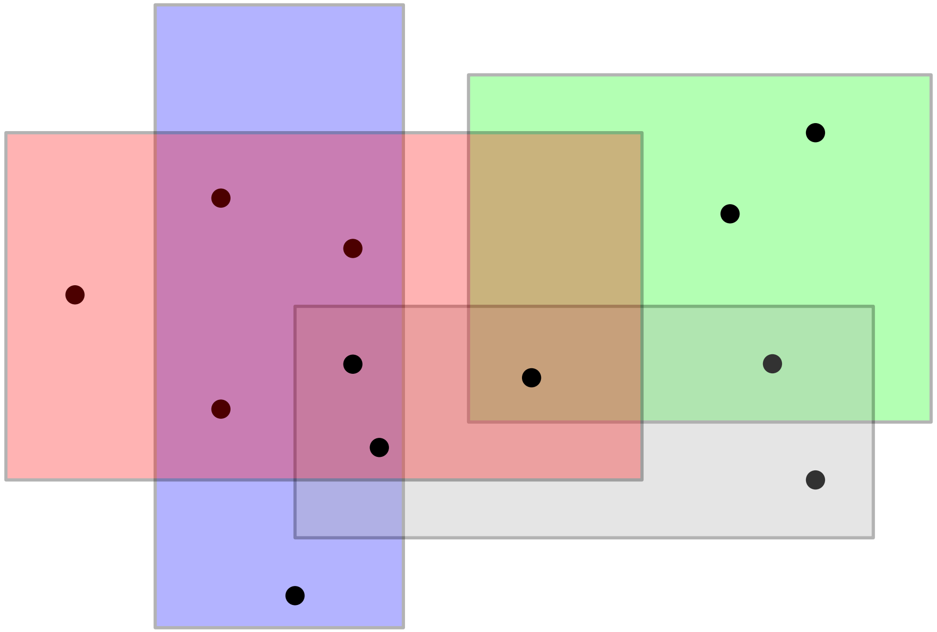
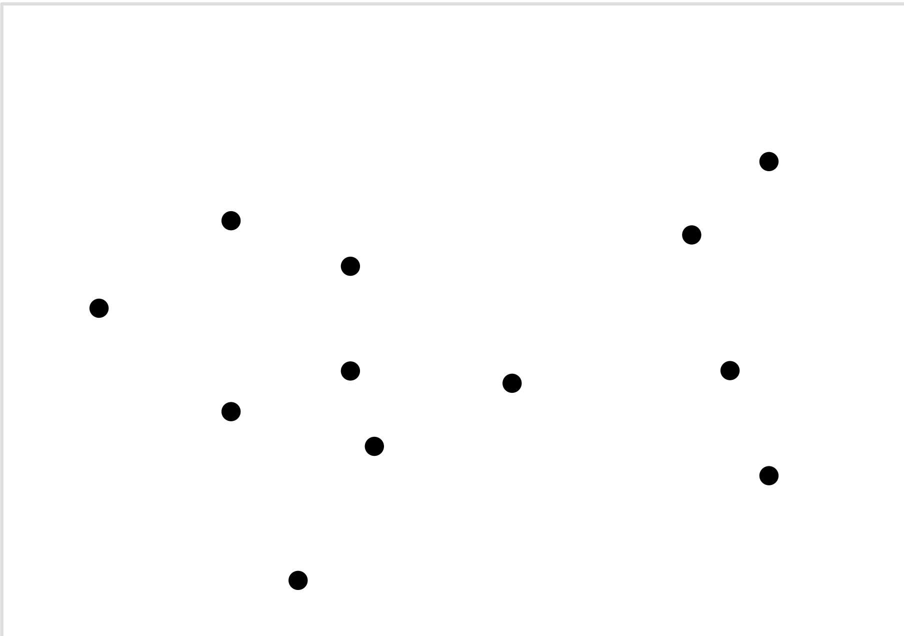


The Maximum Exposure Problem

Neeraj Kumar, Stavros Sintos, Subhash Suri

University of California, Santa Barbara **Duke University**

Problem Description



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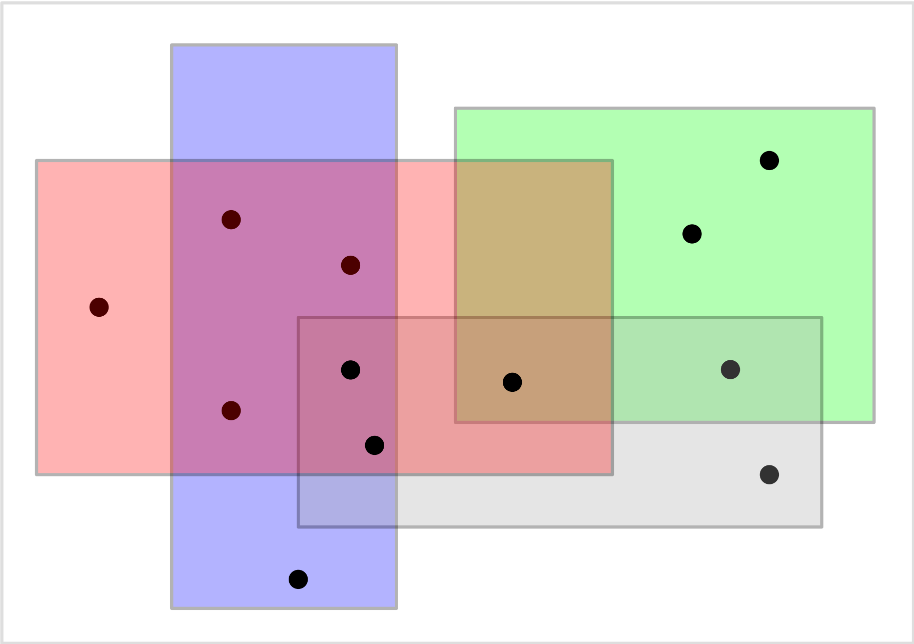

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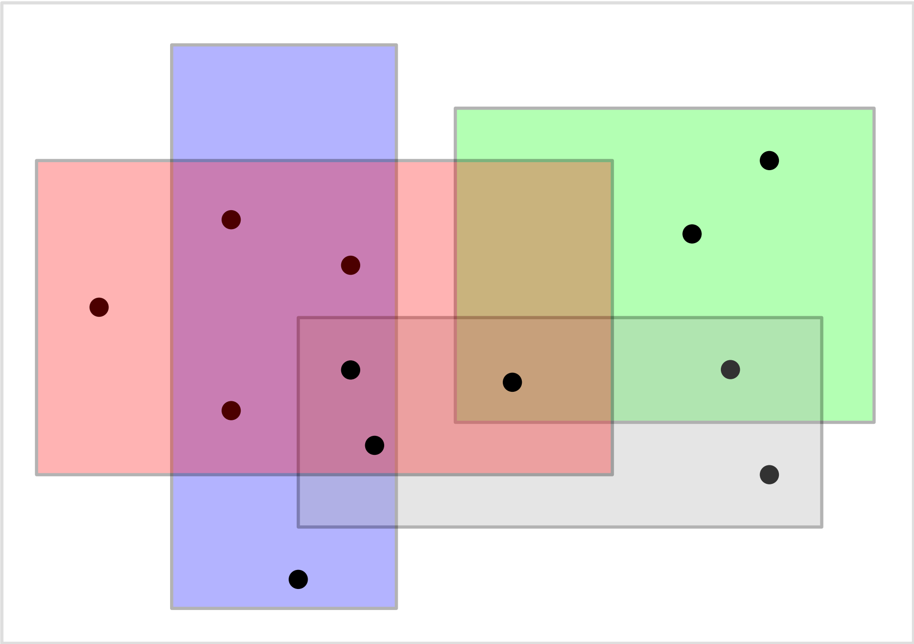

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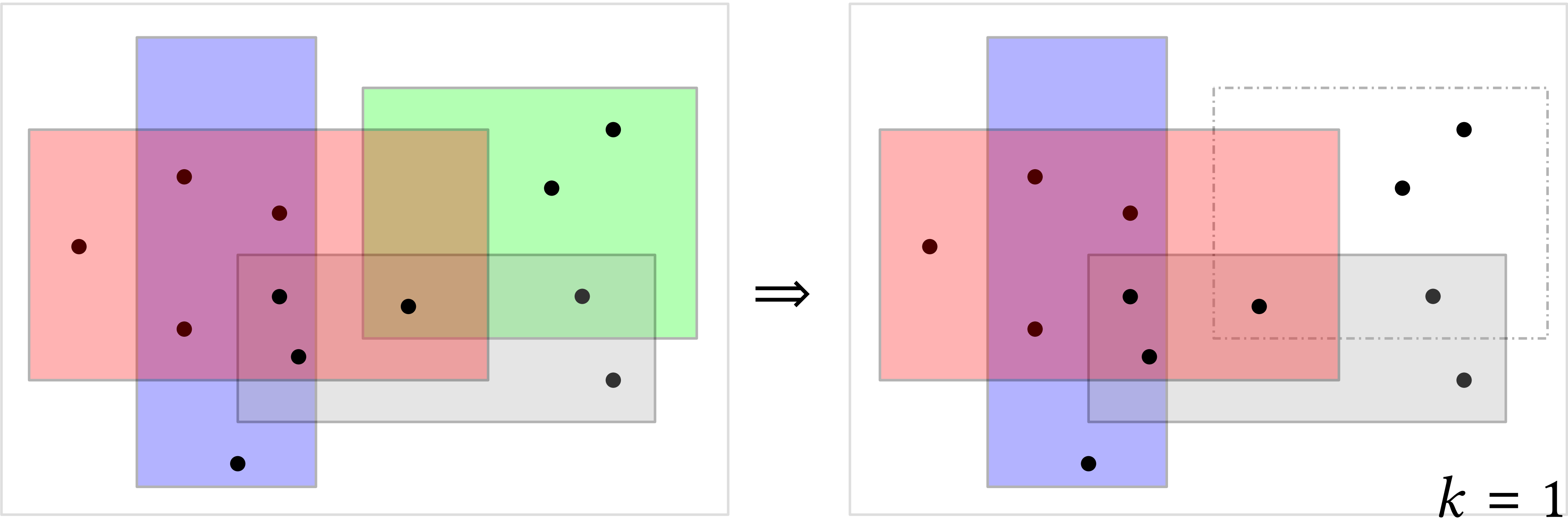

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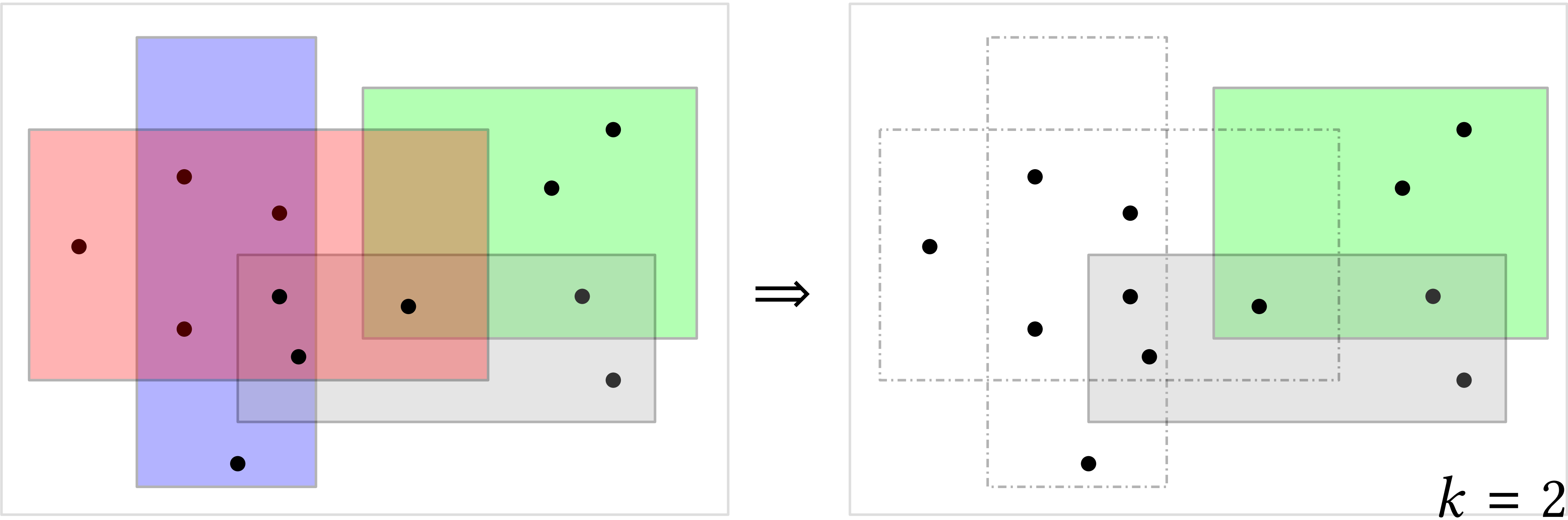

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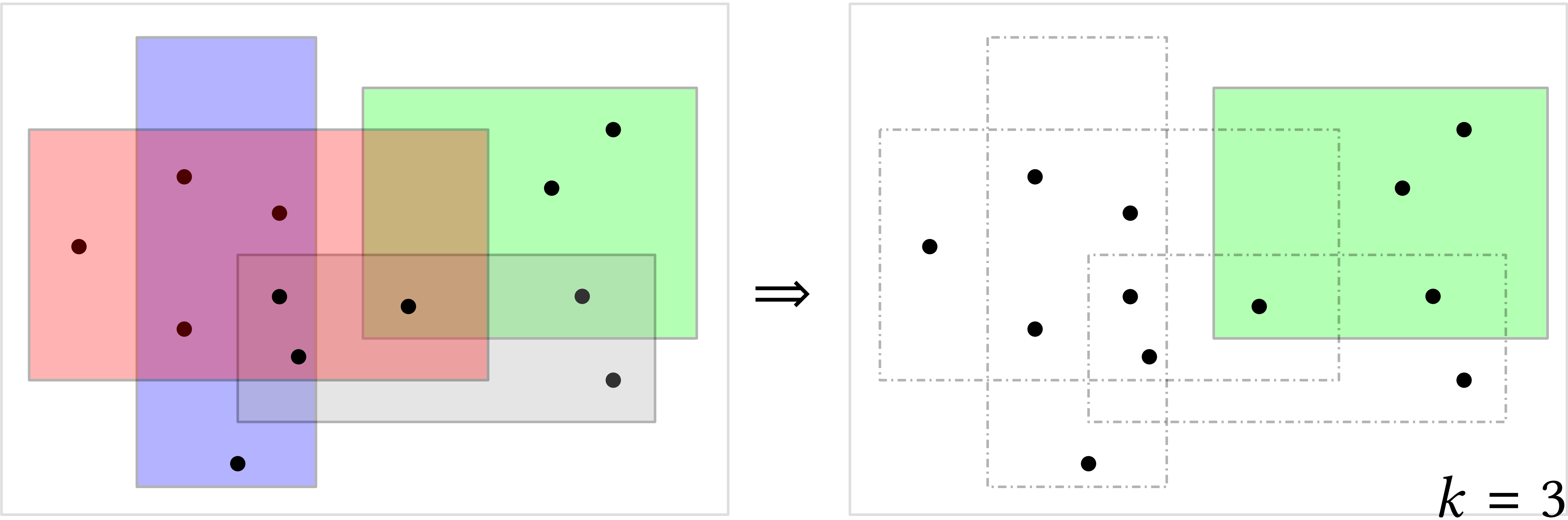

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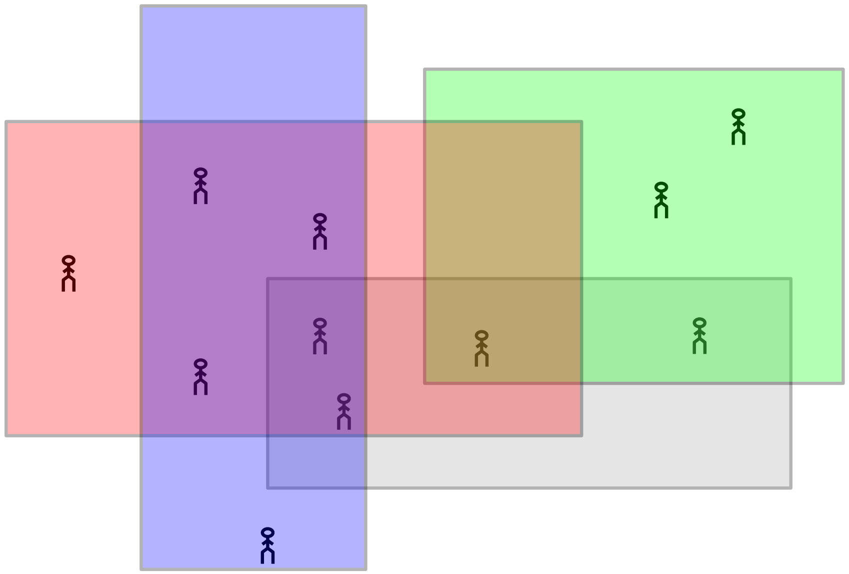
Motivation

- **Reliability of coverage:** points correspond to clients, ranges correspond to coverage of facilities



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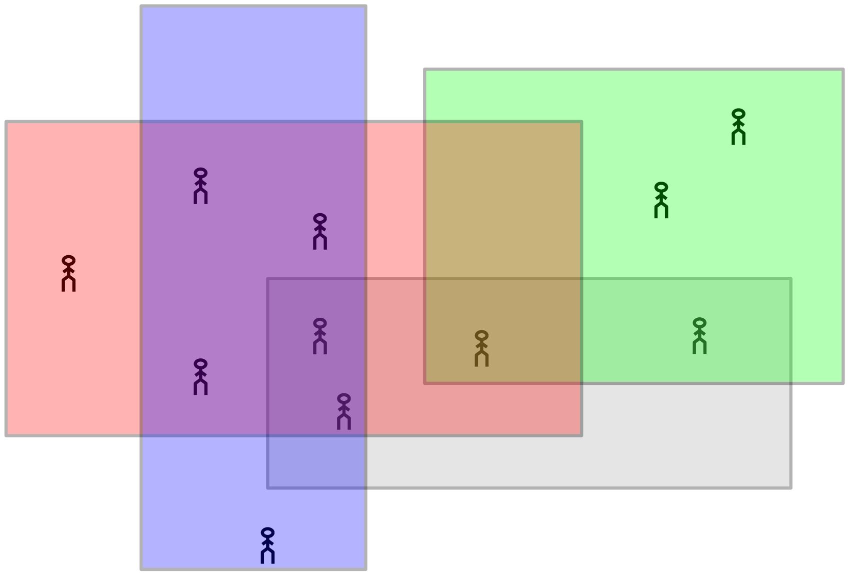

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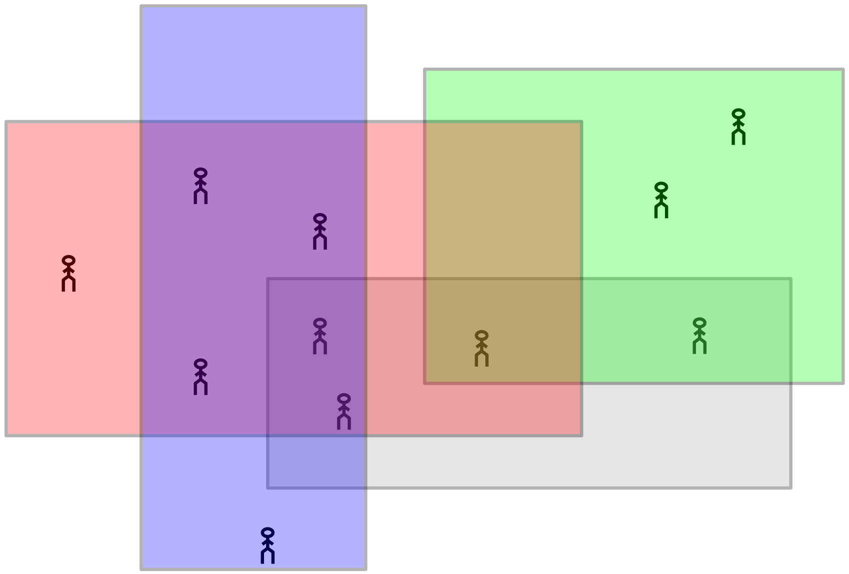

Which k facilities to disable so as to affect maximum number of clients?

- **Geometric constraint removal:** ranges correspond to *constraints*, points correspond to *rewards*

Maximize rewards by removing at most k constraints

Hardness of Max Exposure

- Geometric counterpart of the *densest k-subhypergraph* problem
  – studied recently in (APPROX'16, SODA'17), conditionally hard to approximate within $|V|^{1-\epsilon}$

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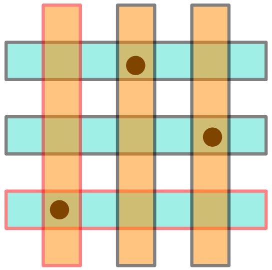

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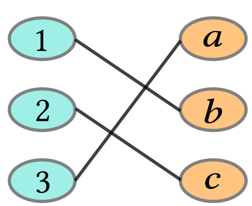

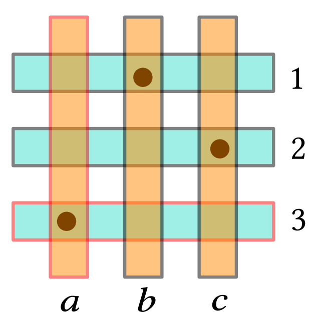

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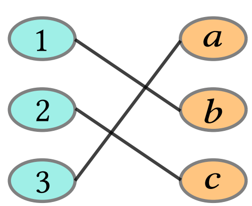

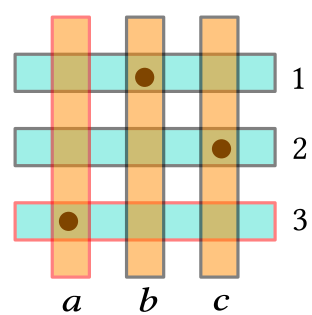

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Simple reduction from densest k-subgraph on bipartite graphs *(bipartite-DkS)*

- Assuming Dense Vs Random conjecture, bipartite-DkS is hard to approximate within $O(|V|^{1/4})$

Approximation Algorithms

Can we do somewhat better for arbitrary rectangles?

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- A bicriteria $O(k)$-approximation for arbitrary rectangles
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  - Gives a constant approximation if ratio of smallest and longest sidelengths is bounded

rest of this talk

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Holds for any polygon with $O(1)$ complexity

Translates of a Single Rectangle

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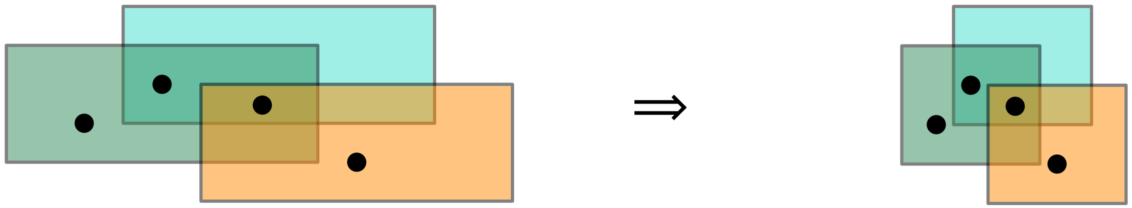

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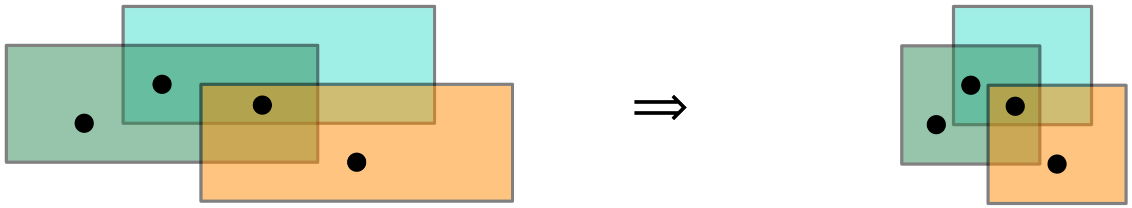

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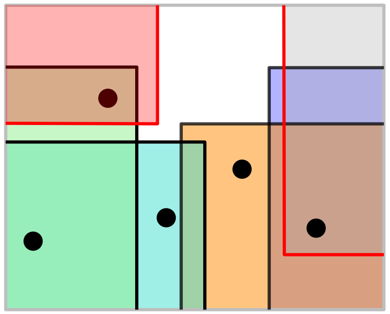

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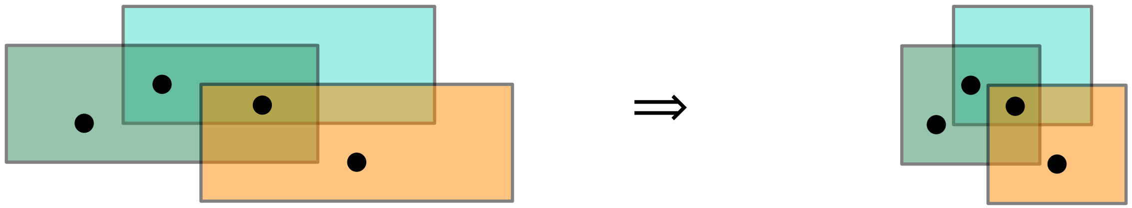

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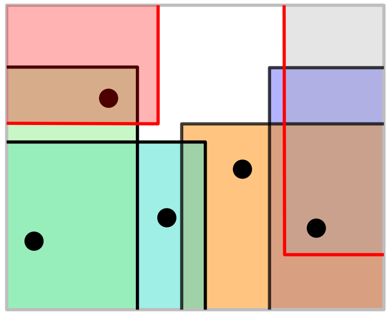

Roadmap

Within a unit square $\rightarrow$ Within a horizontal strip of unit width $\rightarrow$ PTAS

(polytime) (polytime) (shifting techniques)

$\Longrightarrow$ 4-approximation $\Longrightarrow$ 2-approximation

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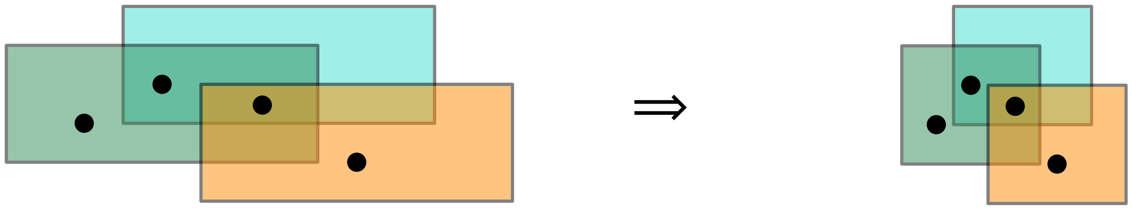

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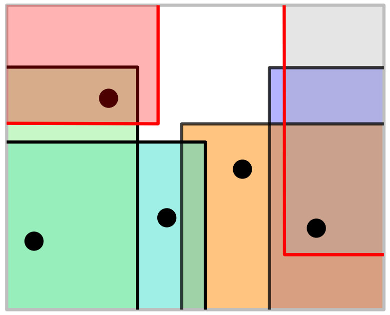

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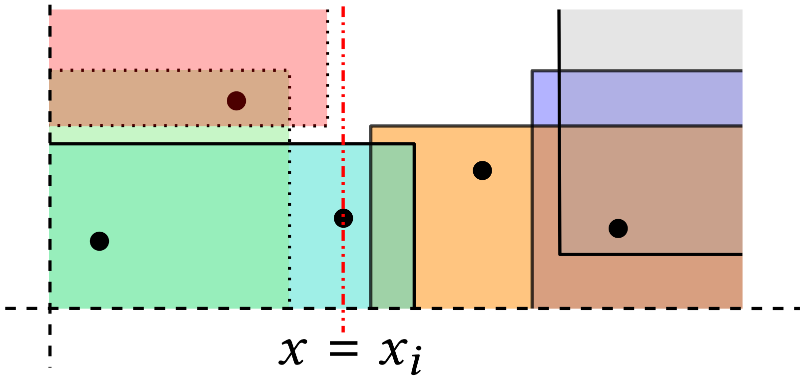

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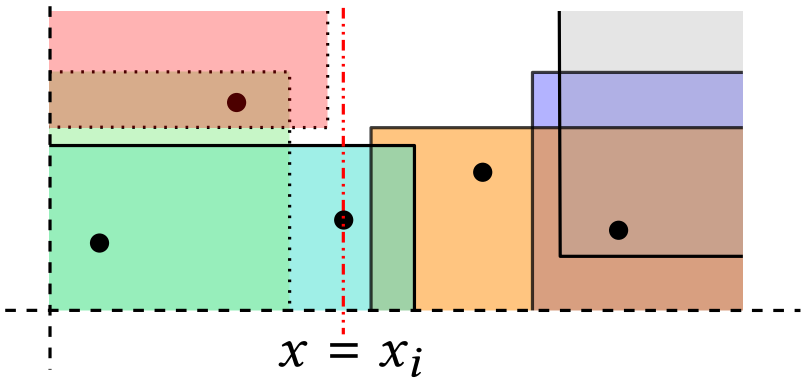

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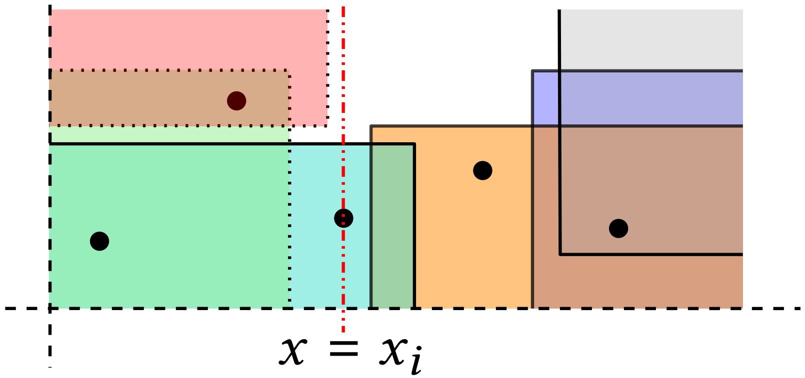

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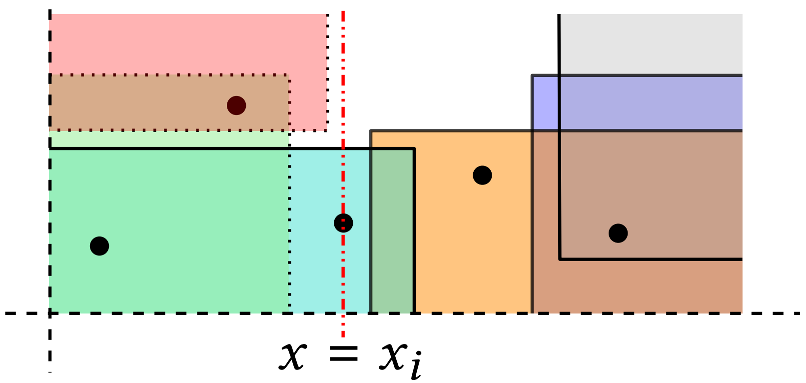

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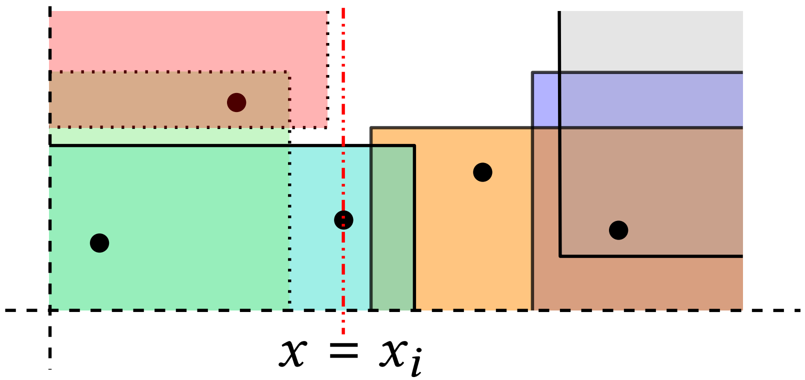

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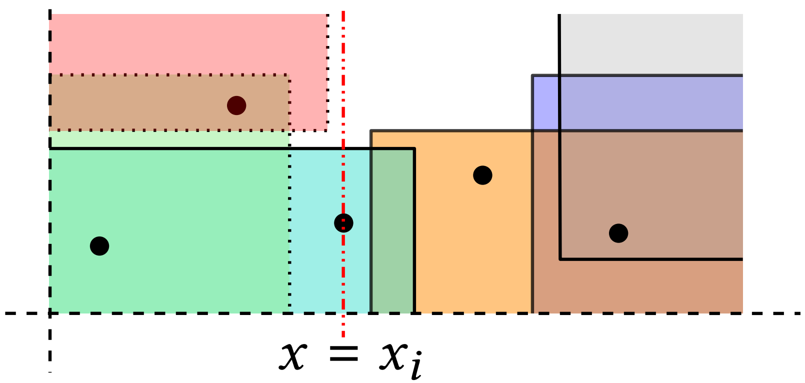

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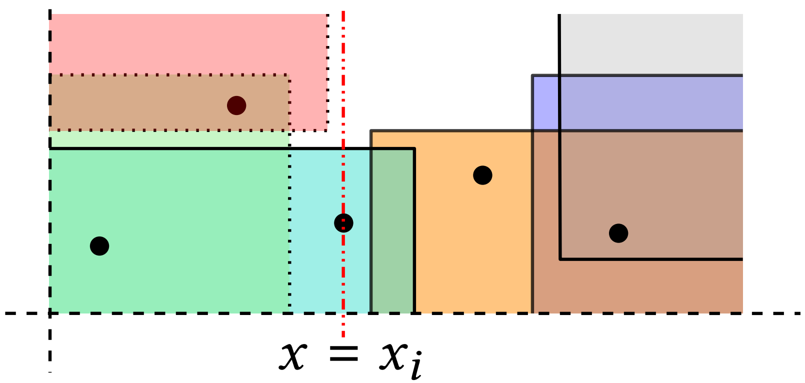

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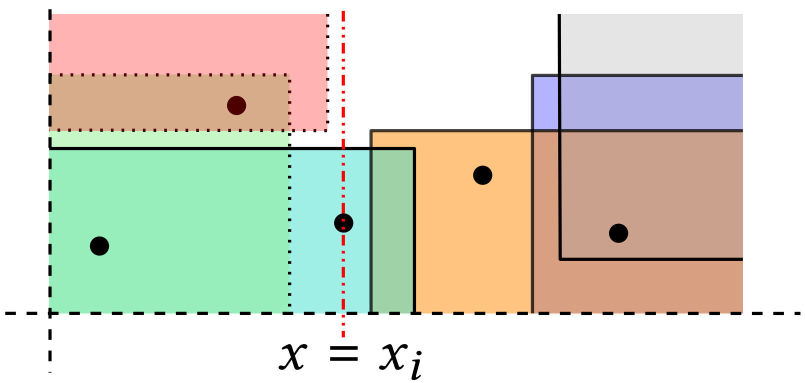

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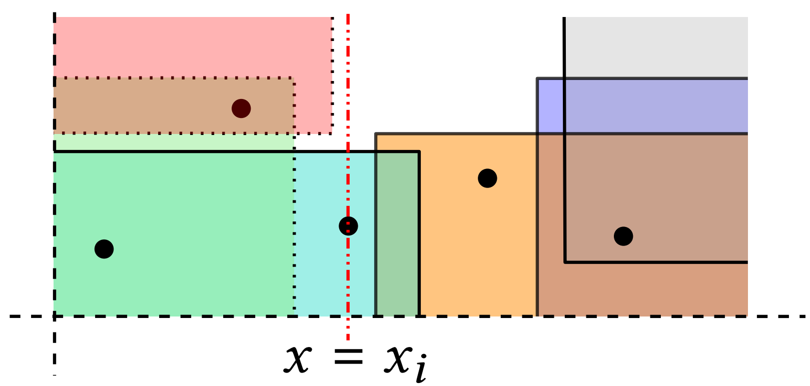

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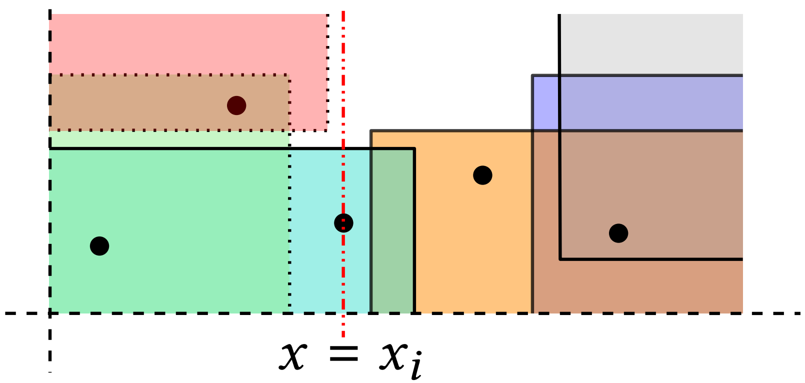

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$$S(i,\ k', \mathcal{R}_d) = \max \begin{cases} S(i+1,\ k',\ \mathcal{R}_d) & \text{do not expose } p_i \\ S(i+1,\ \boxed{k' - k_i,}\ \mathcal{R}_d \cup \mathcal{R}(p_i)) \ + \ 1 & \text{expose } p_i \end{cases}$$

$k_i = |\mathcal{R}(p_i) \setminus \mathcal{R}_d|$

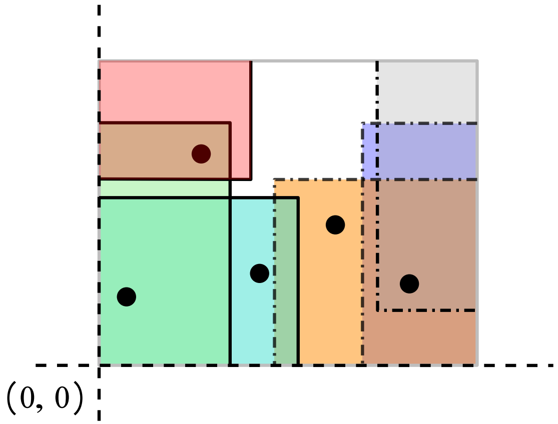
Set of active ranges that were already deleted

\# of ranges that can be deleted to right of $x = x_i$ $(0 \le k' \le k)$

Optimal solution : $S(0,\ k,\ \varnothing)$

Max-Exposure Within a Unit Square

How do we keep track of deleted range set $\mathcal{R}_d$ using polynomial space?



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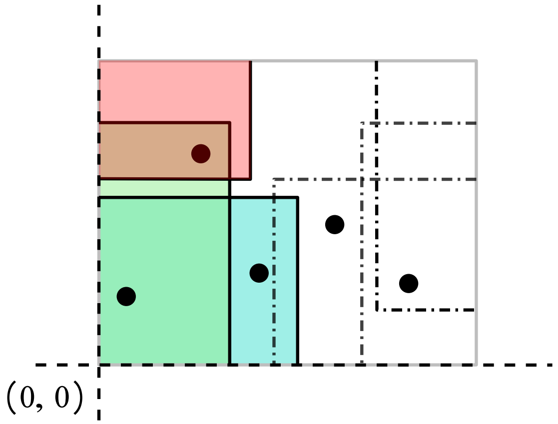

Type-0: Unit square ranges that intersect $x = 0$

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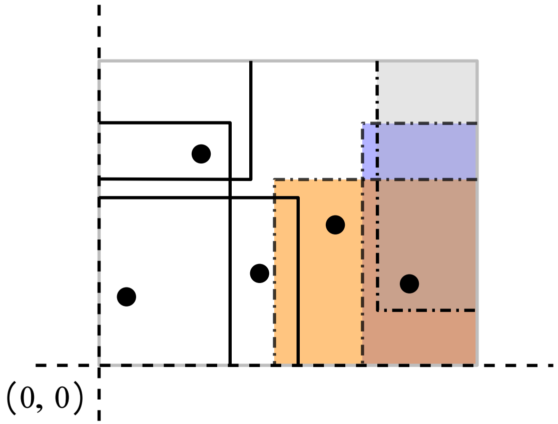

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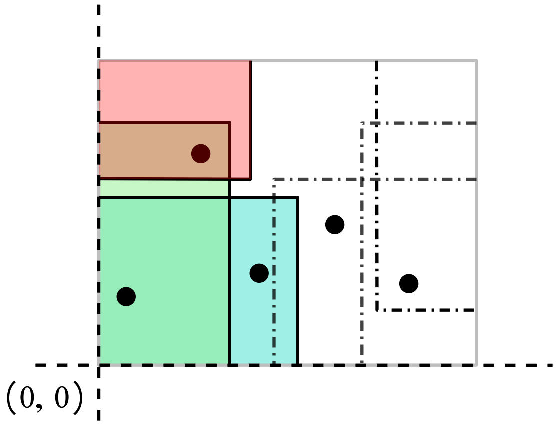

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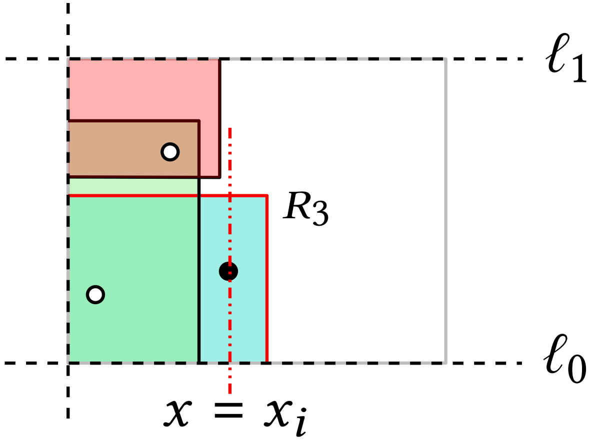
Suppose we only had Type-0 ranges:



R_3 is 'anchored' to ℓ_0

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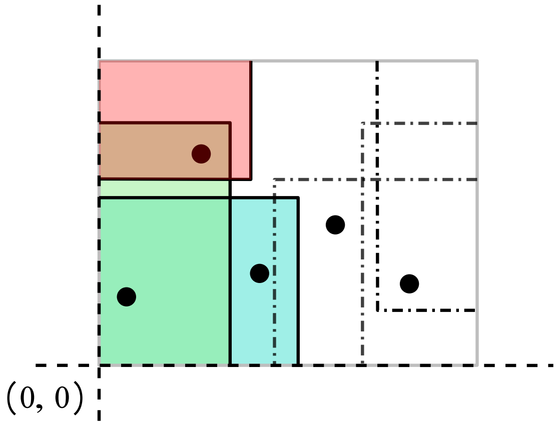

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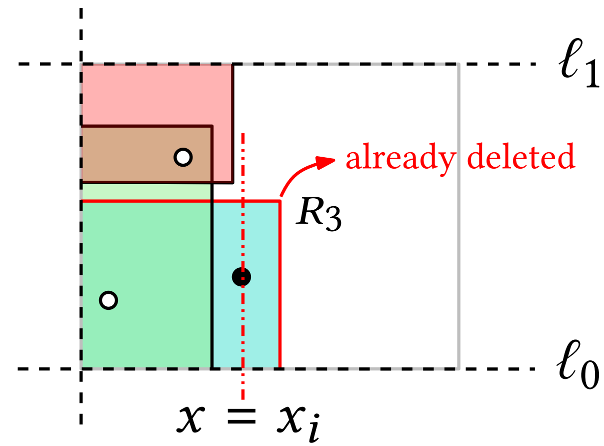

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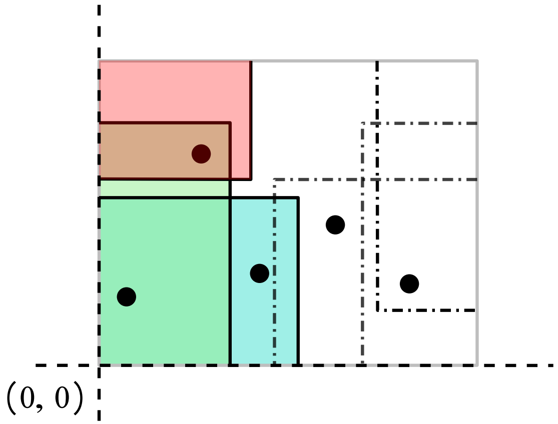

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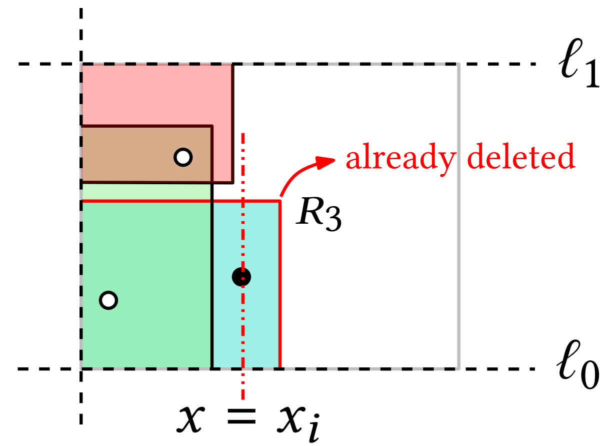

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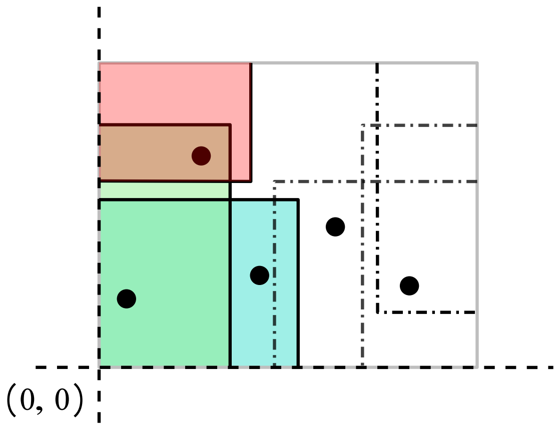

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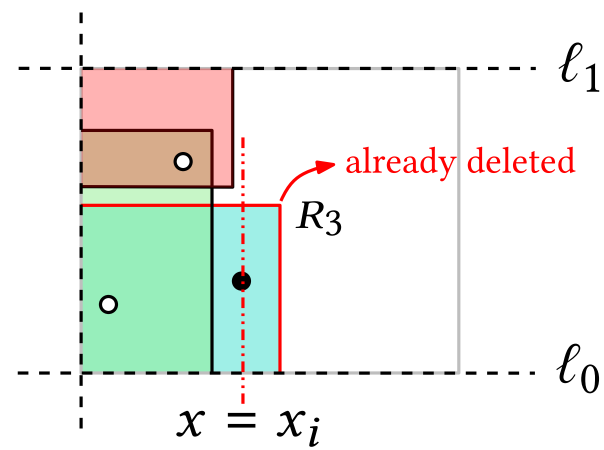

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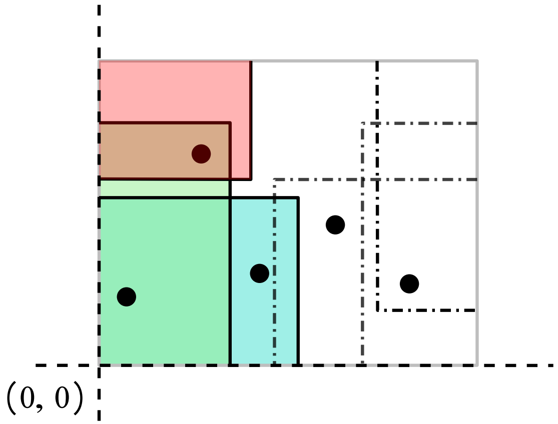

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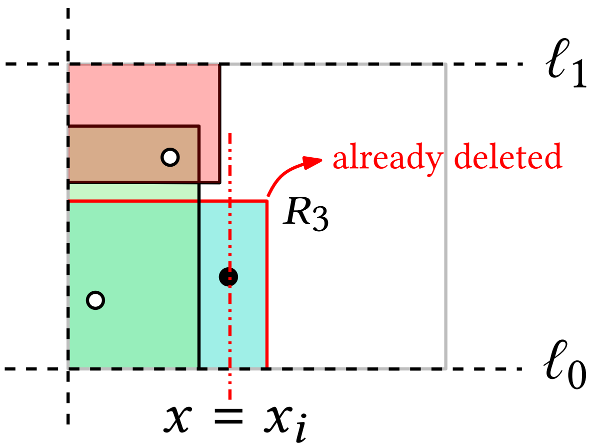

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R_3 is 'anchored' to ℓ_0
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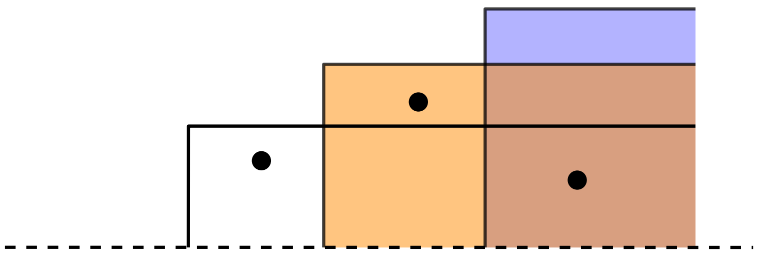
Can keep track of Type-0 deleted ranges by remembering q_0, q_1

Handling Type-1 Ranges

Need an alternative dynamic programming formulation : **DP-template-1**



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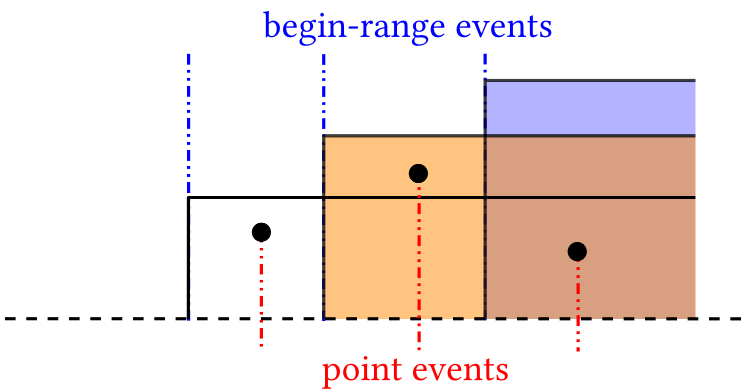

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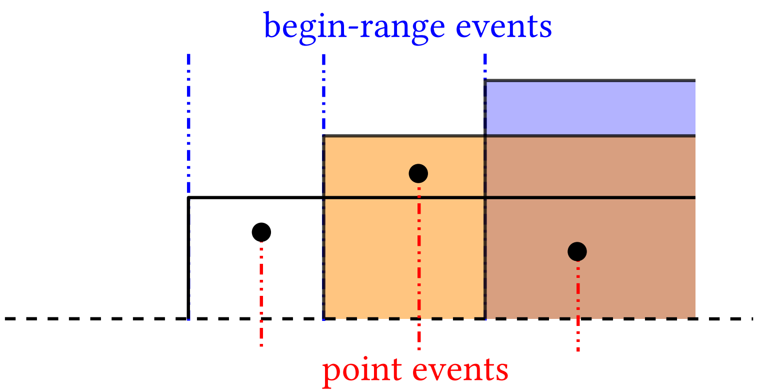

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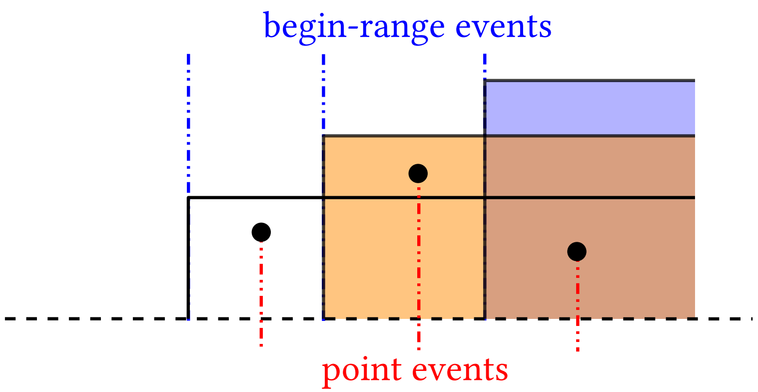

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Maintain set of *forbidden points* P_f

active points that lie in a range that was not deleted

$S(i,\ k',P_f\ )$

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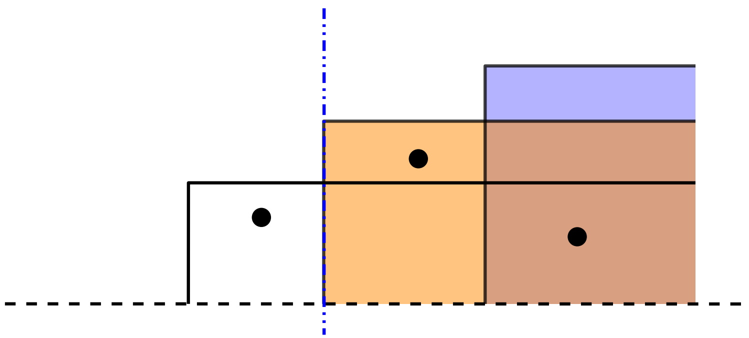

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$$S(i,\ k', P_f)\ =\ \max\begin{cases} S(i+1,\ k'-1,\ P_f) & \text{delete range } R_i \\ S(i+1,\ k',\ P_f \cup P(R_i)) & \text{do not delete } R_i \end{cases}$$

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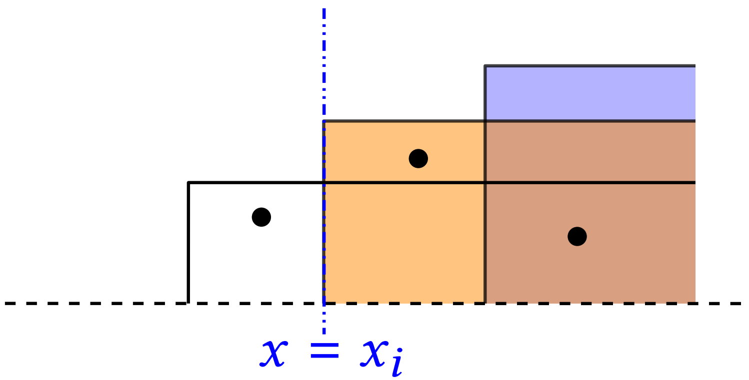

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All points contained in R_i

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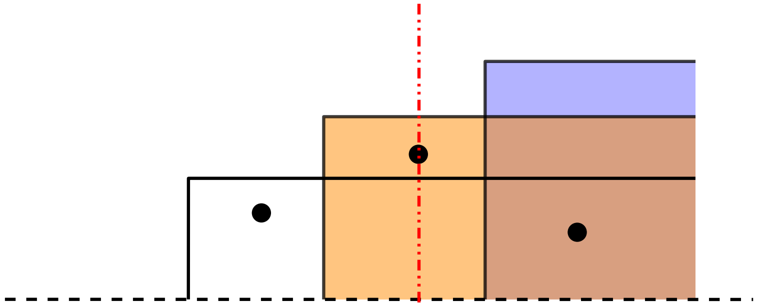

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$$= \max \begin{cases} S(i+1,\ k', P_f) & \text{if } p_i \in P_f\text{, cannot expose } p_i \\ S(i+1,\ k', P_f) + 1 & \text{otherwise, expose } p_i \end{cases}$$

Point p_i

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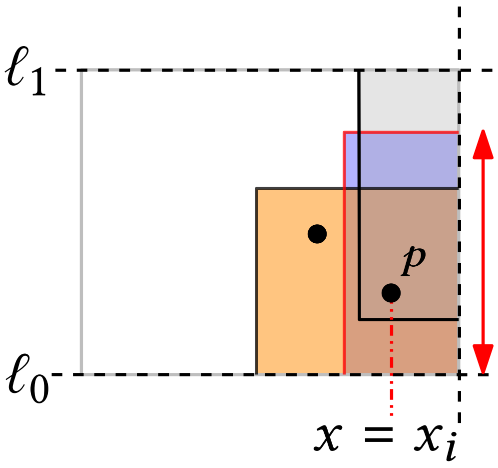
Handling Type-1 Ranges

How do we keep track of forbidden points P_f using polynomial space?

Q_0 = Undeleted range to left of $x = x_i$ farthest from ℓ_0



Handling Type-1 Ranges

How do we keep track of forbidden points P_f using polynomial space?

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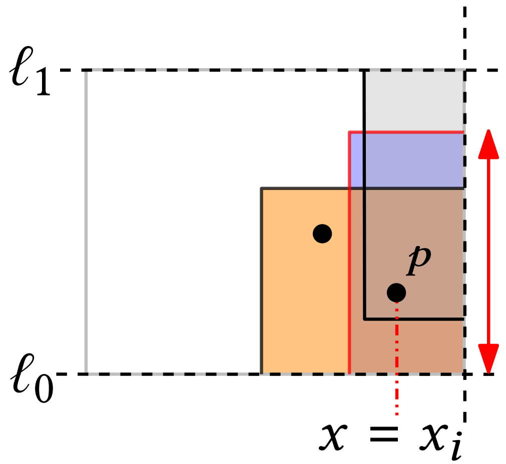
$$P_f = P(Q_0) \cup P(Q_1)$$



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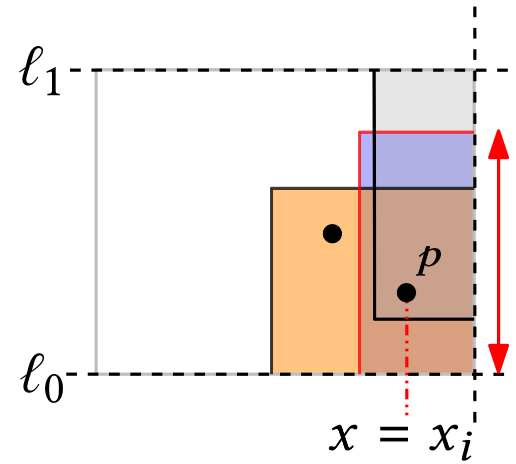

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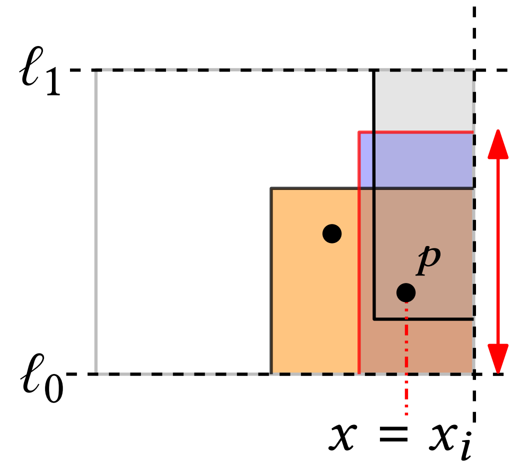

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Combine **DP-template-0** and **DP-template-1** to solve wihin a unit square:

Subproblems defined as : $S(i,\ k',\ \underline{q_0,\ q_1,\ Q_0,\ Q_1})$

updated appropriately at begin-range and point events

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Thanks!

Backup: Combined DP

$S(i,\ k',\ q_0,\ q_1, Q_0, Q_1)$

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$$= \max \begin{cases} S(i+1,\ k' - 1,\ q_0,\ q_1,\ Q_0,\ Q_1) & \text{delete Type-1 range } R_i \\ S(i+1,\ k',\ q_0,\ q_1,\ \mathit{farther}(R_i, Q_0),\ Q_1) & R_i \text{ is not deleted and anchored to } \ell_0 \\ S(i+1,\ k',\ q_0,\ q_1,\ Q_0,\ \mathit{farther}(R_i, Q_1)) & R_i \text{ is not deleted and anchored to } \ell_1 \end{cases}$$

Point p_i