Shortest Paths in the Plane with Obstacle Violations

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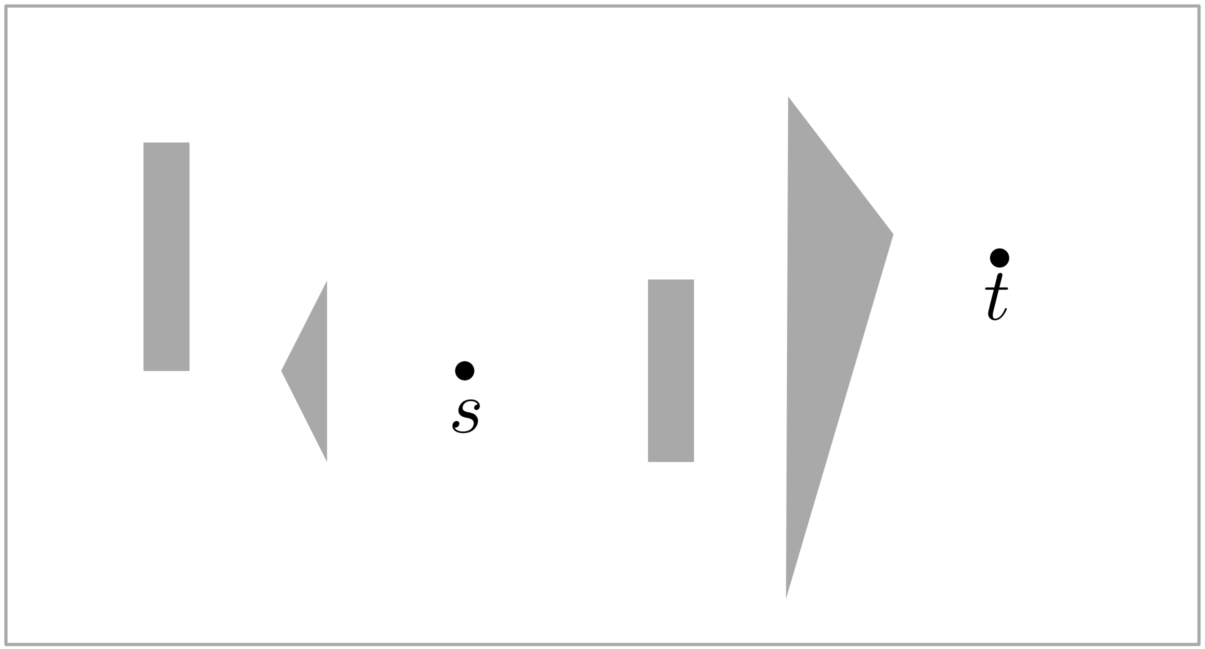
[*] Mentor Graphics Corporation

[†] University of California, Santa Barbara

Problem Description

Input: A polygonal domain P with h convex obstacles, n vertices, source s and target t



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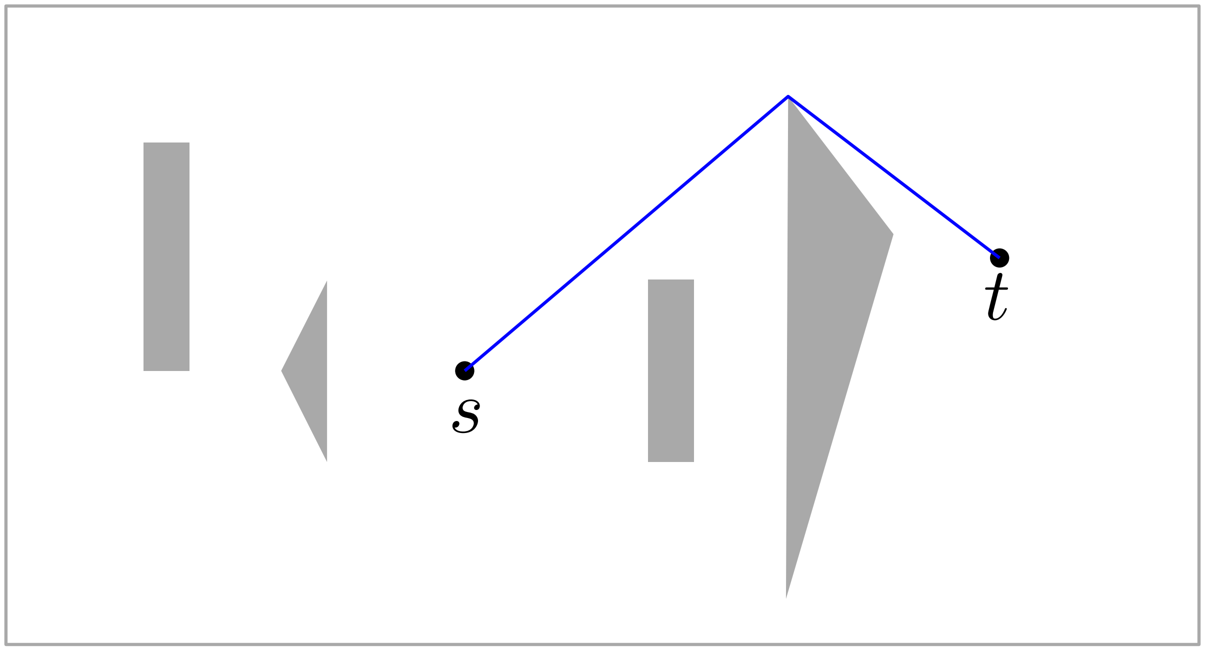

Classical Shortest Path Problem : Obstacles Impenetrable

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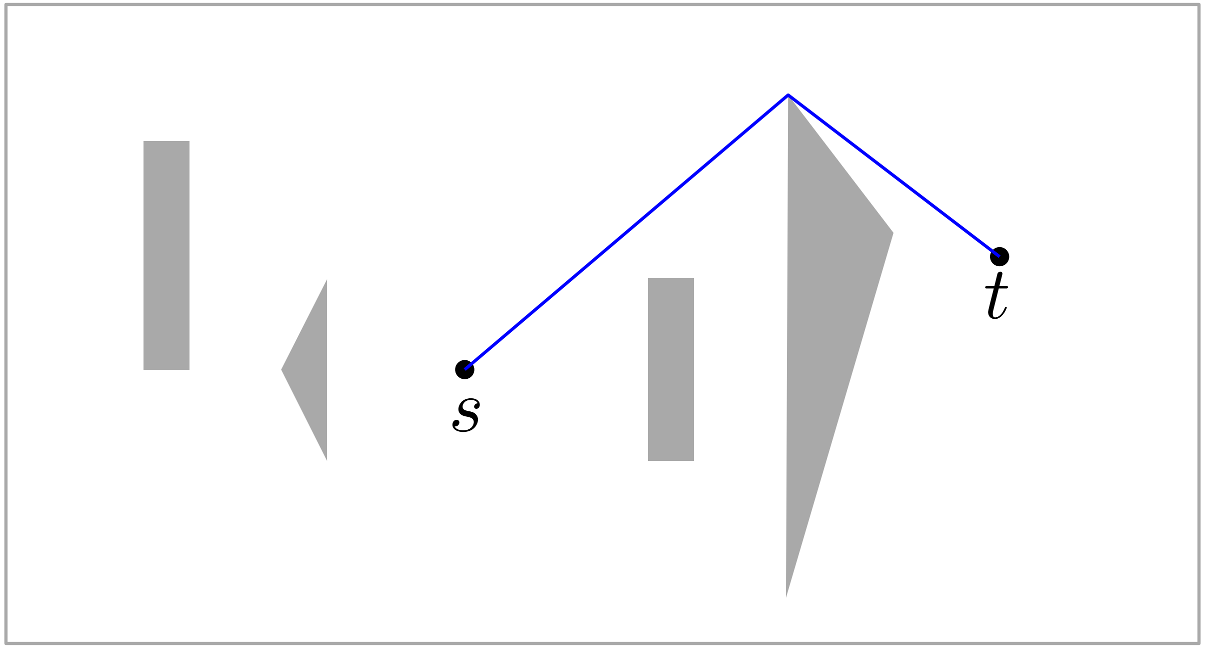

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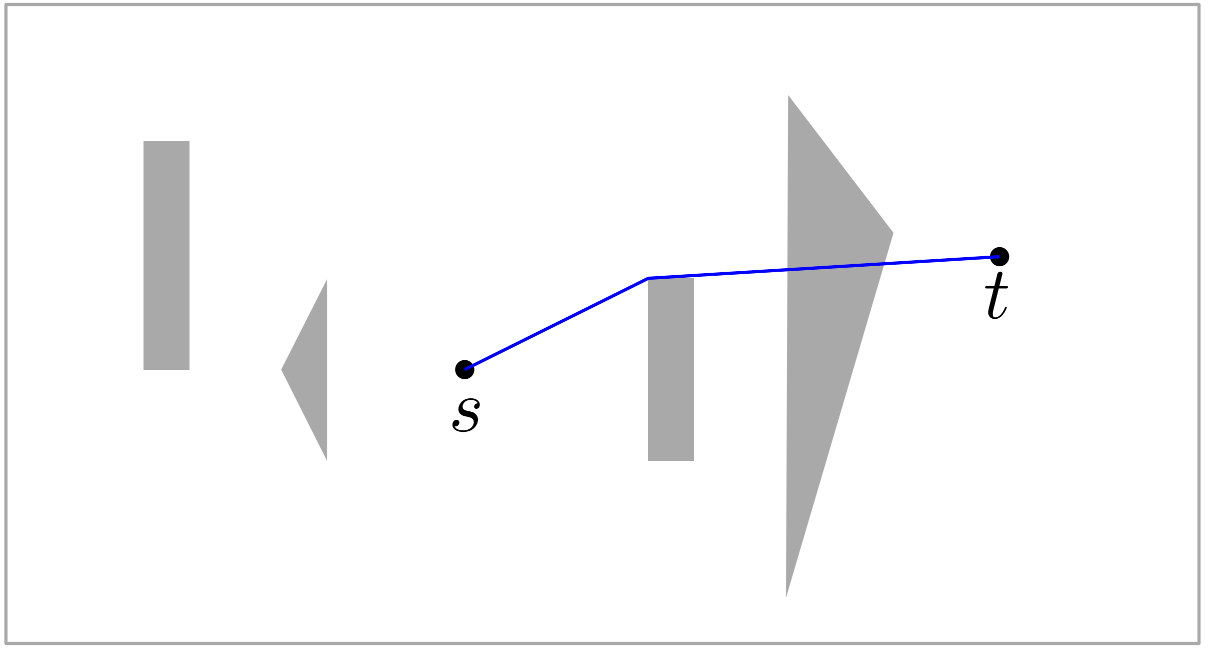

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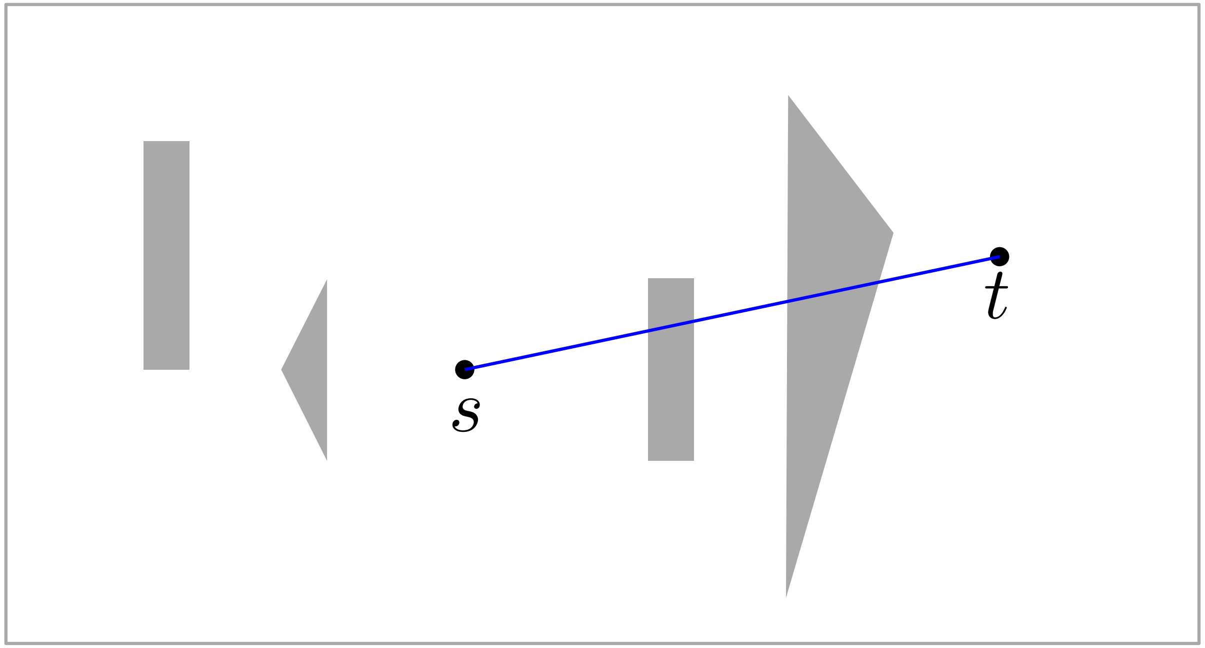

Allowed to cross two obstacles

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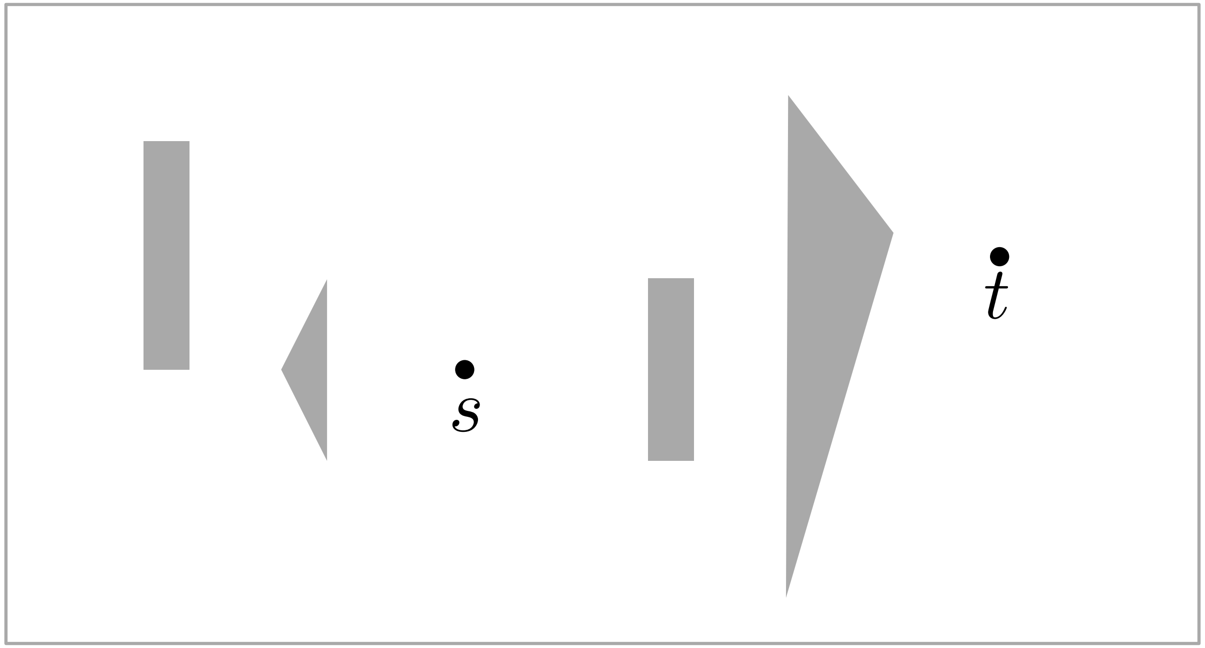

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Compute shortest s–t path that can go through at most k obstacles

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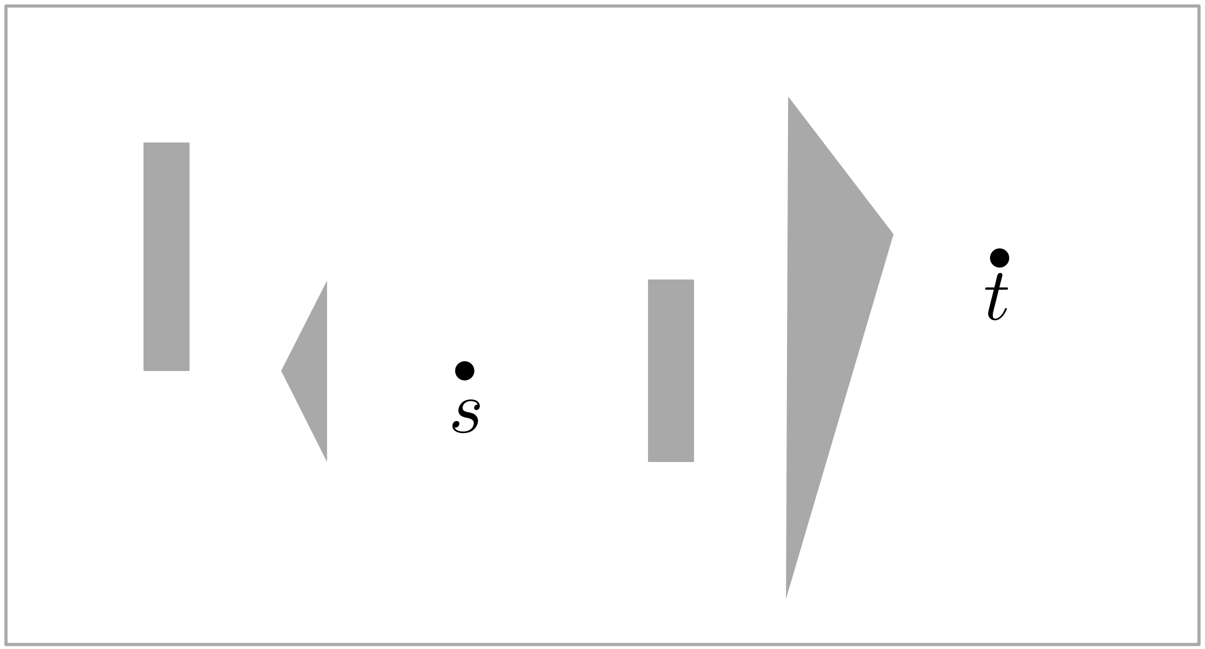

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k-path π_k

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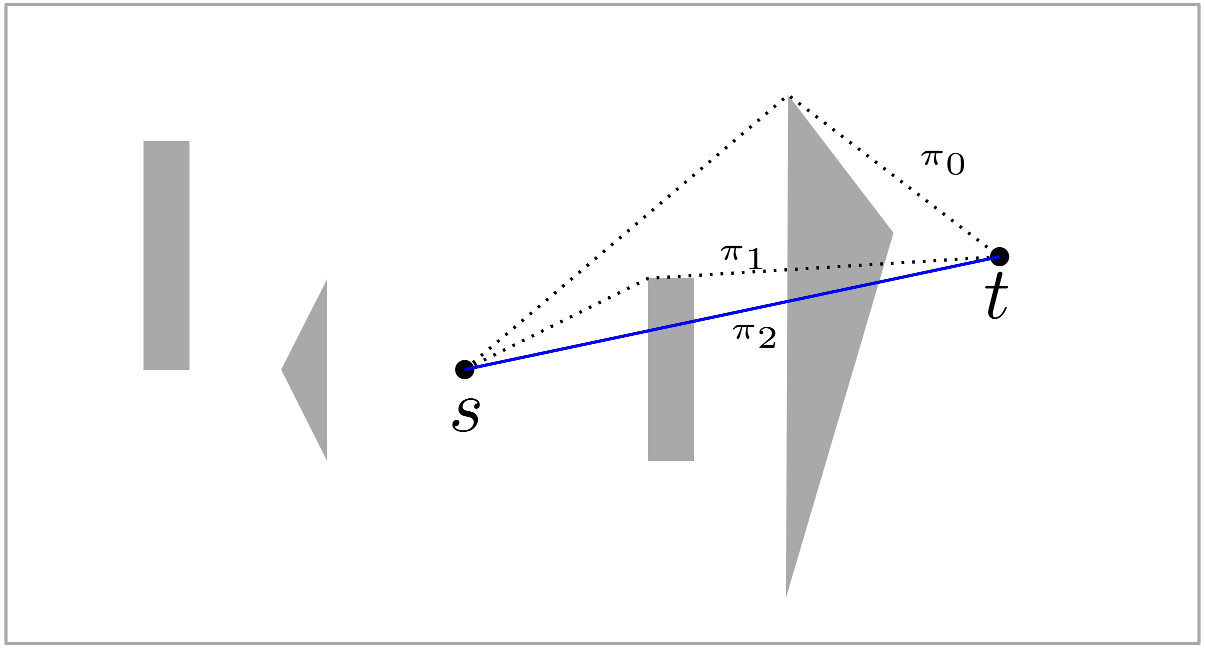

Shortest k-paths

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Shortest k-Paths : Motivation

A ‘less restricted’ version of classical shortest path problem

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Lots of Related work

Visibility Graph [GM '91, KM'88. . .]

Continuous Dijkstra [HS '97, RS'94, M'87 . . .]

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$\Rightarrow$ Robot Motion Planning

For example , obstacles are doors that can be opened

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Paying for a toll bridge vs a longer route?

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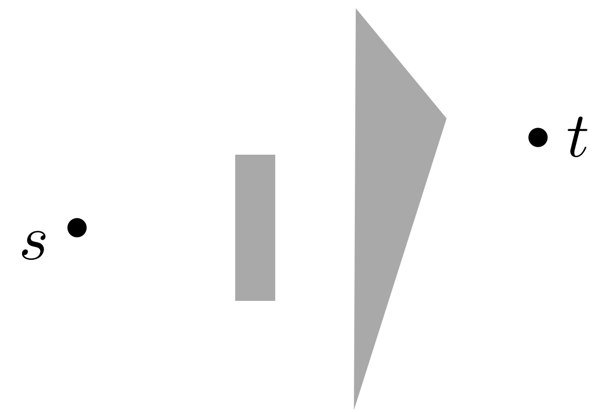
Paying for a toll bridge vs a longer route?

$\Rightarrow$ Geometric Network Augmentation

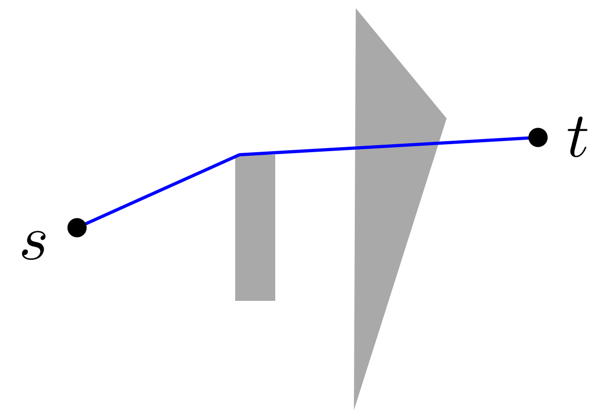
Removing Obstacles $\equiv$ Adding Edges to Visibility Graph

A Simple $O(kn^2)$ Algorithm



A Simple $O(kn^2)$ Algorithm

Key Idea: Shortest k-path turns only at obstacle vertices



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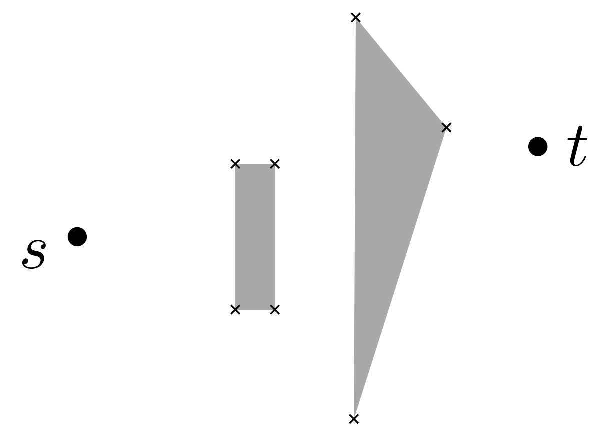
Construct k-visibility graph G_k



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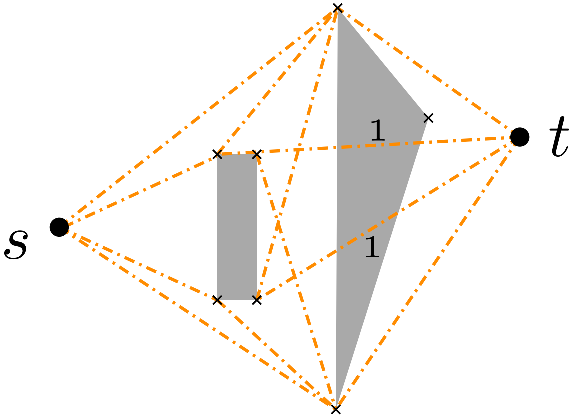
- $\overline{pq}$ is an edge if it crosses $\leq k$ obstacles
- Label edges with crossing number
- Weight of an edge is its Euclidean length



$O(n)$ vertices; $O(n + h^2)$ edges

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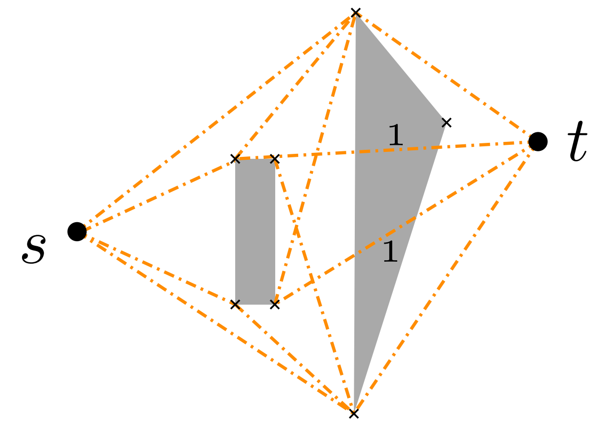

$O(n)$ vertices; $O(n + h^2)$ edges

Find shortest s–t path such that sum of labels on its edges is $\leq k$

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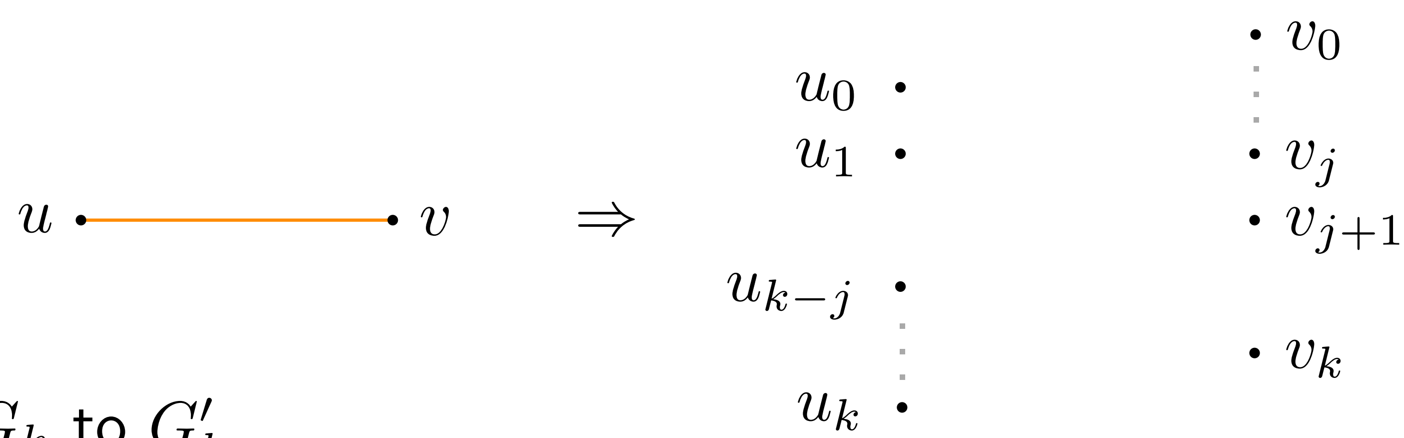

Transform G_k to G'_k

– k copies of each vertex

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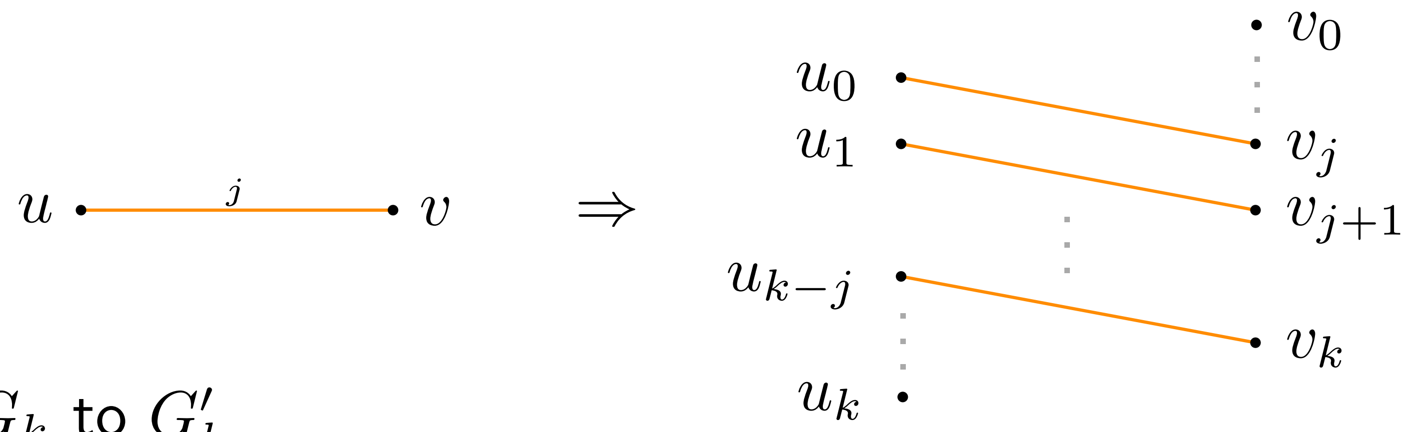

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- For edge (u, v) with j crossings

  Add edges $(u_0, v_j), (u_1, v_{j+1})$ $\ldots$ (u_{k-j}, v_k)
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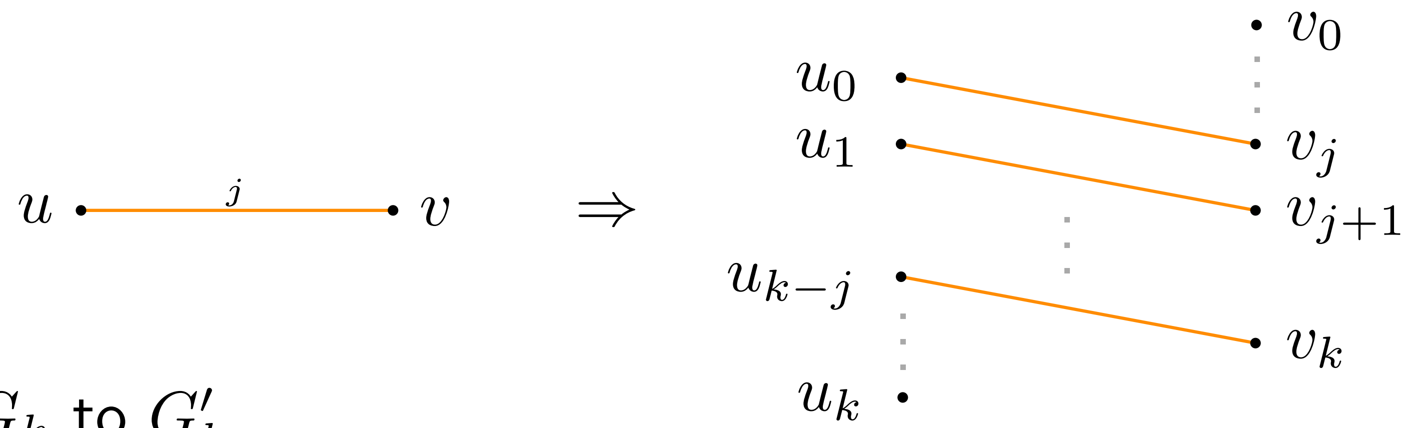

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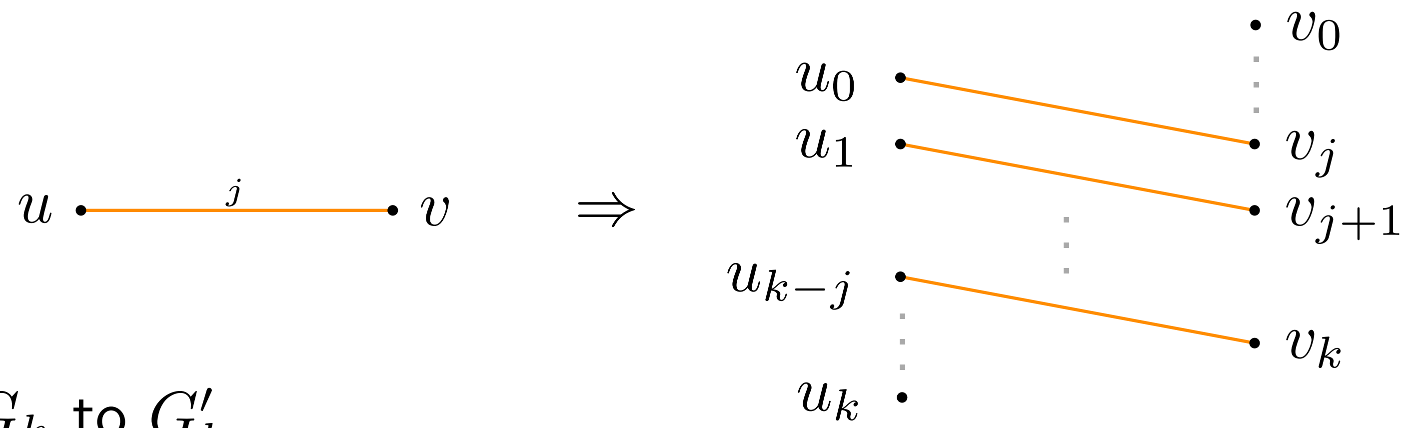

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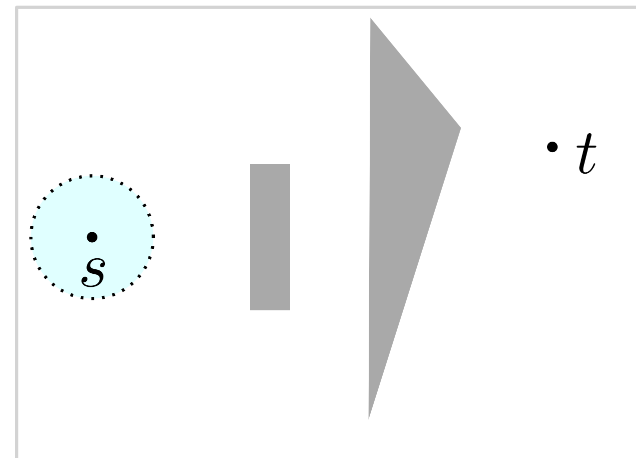
$O(kn^2)$ worst case using Dijkstra's Algorithm

Towards a faster Algorithm : Continuous Dijkstra

Simulates propagation of unit-speed wavefront starting at s



Wavefront at time $T \equiv$ all points at distance T from s

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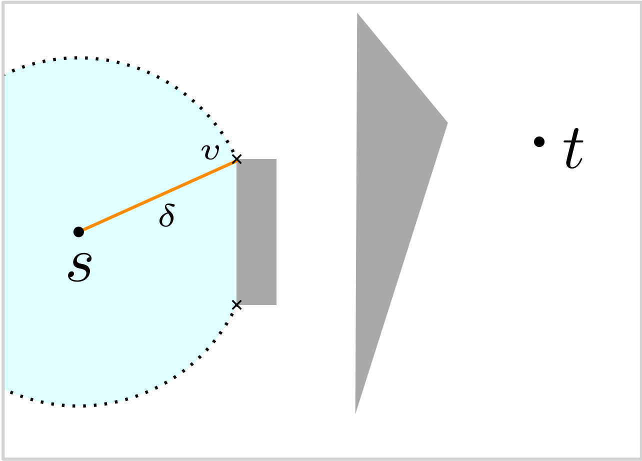

Adds new wavelet identified by its source $(v\,,\,\delta)$

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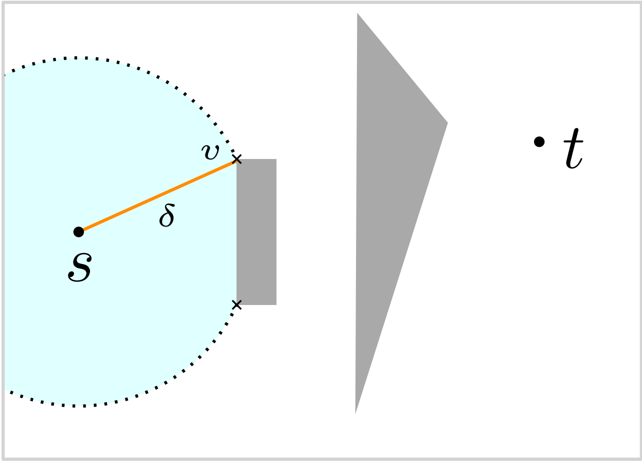

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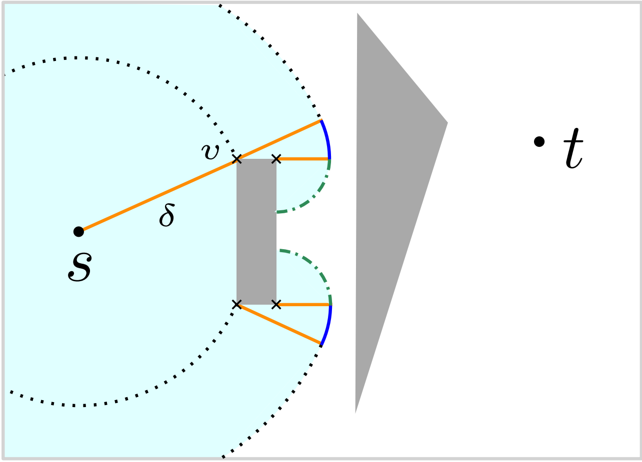

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– Challenge is to keep track of events

Wavelet-Obstacle collisions

Wavelet-Wavelet collisions

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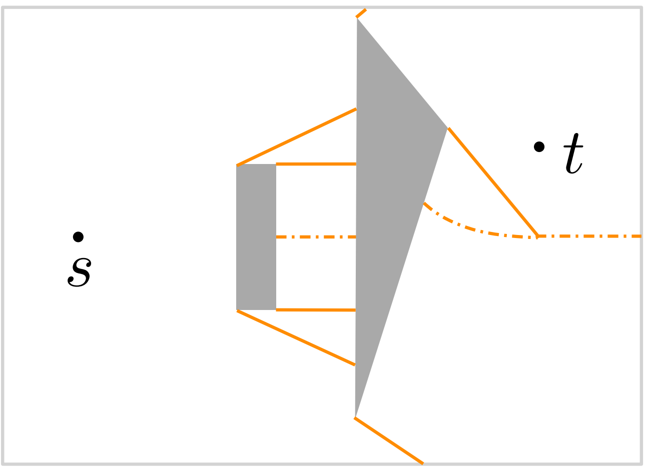

Shortest Path Map (SPM)

- Computes a planar subdivision of P : Shortest Path Map

$O(n \log n)$ algorithm known [Hershberger-Suri, '97]

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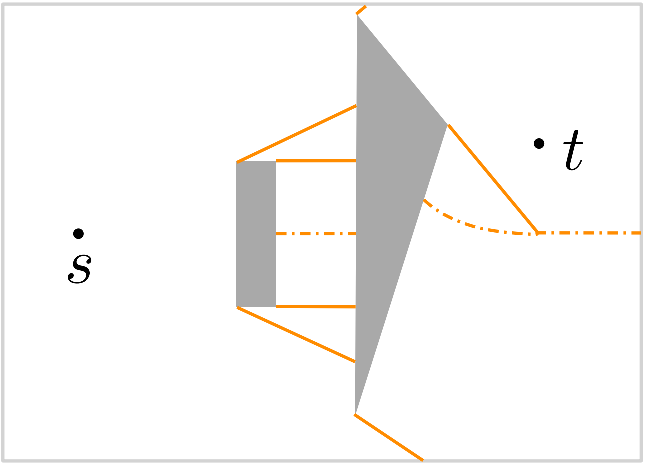

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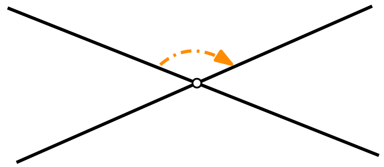

If they did, can locally reconnect to obtain a shorter path

Shortest 0-paths do not cross each other

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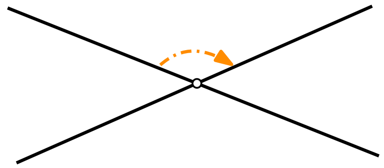

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NOT true for shortest k-paths for $k > 0$

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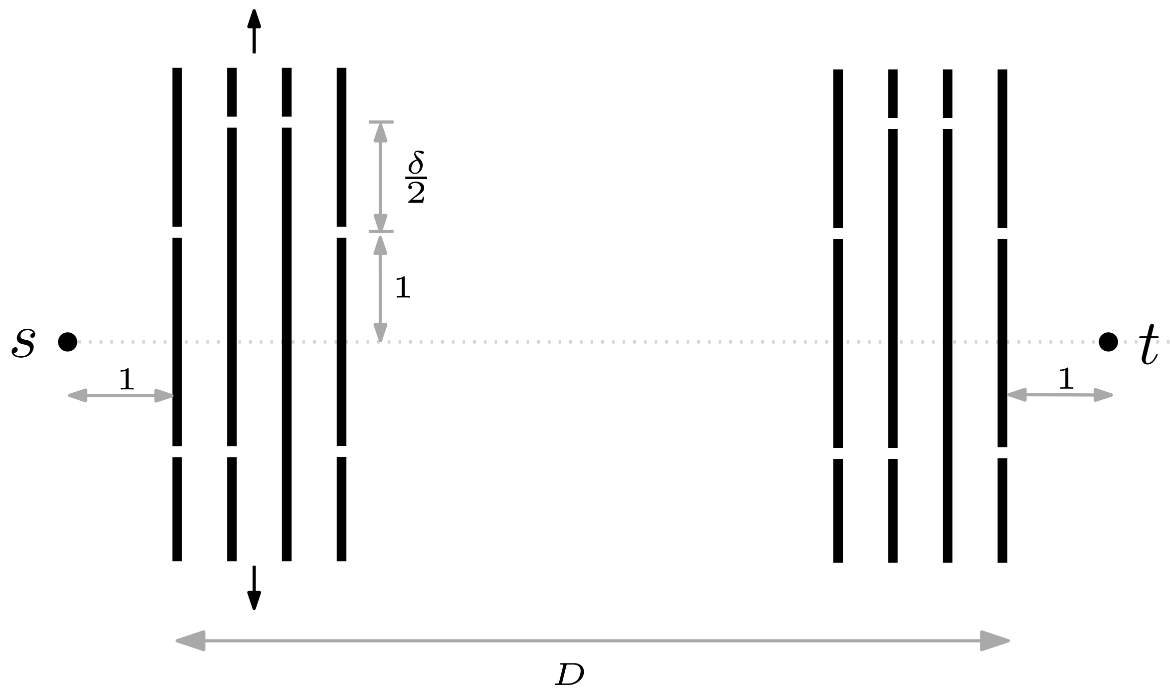
Shortest k-Paths may Cross!



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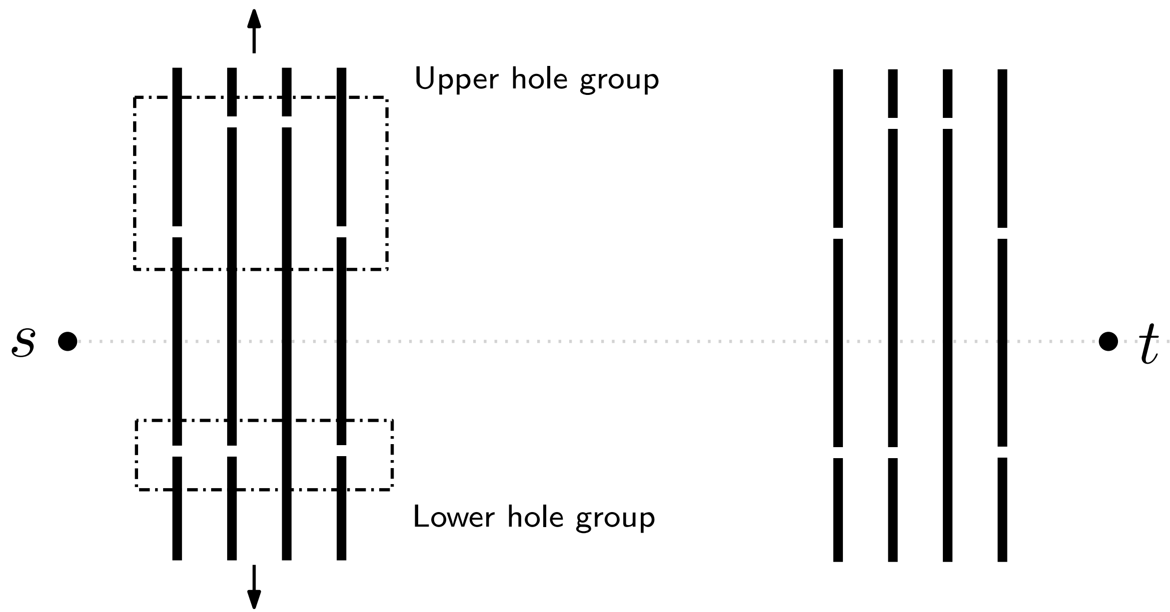

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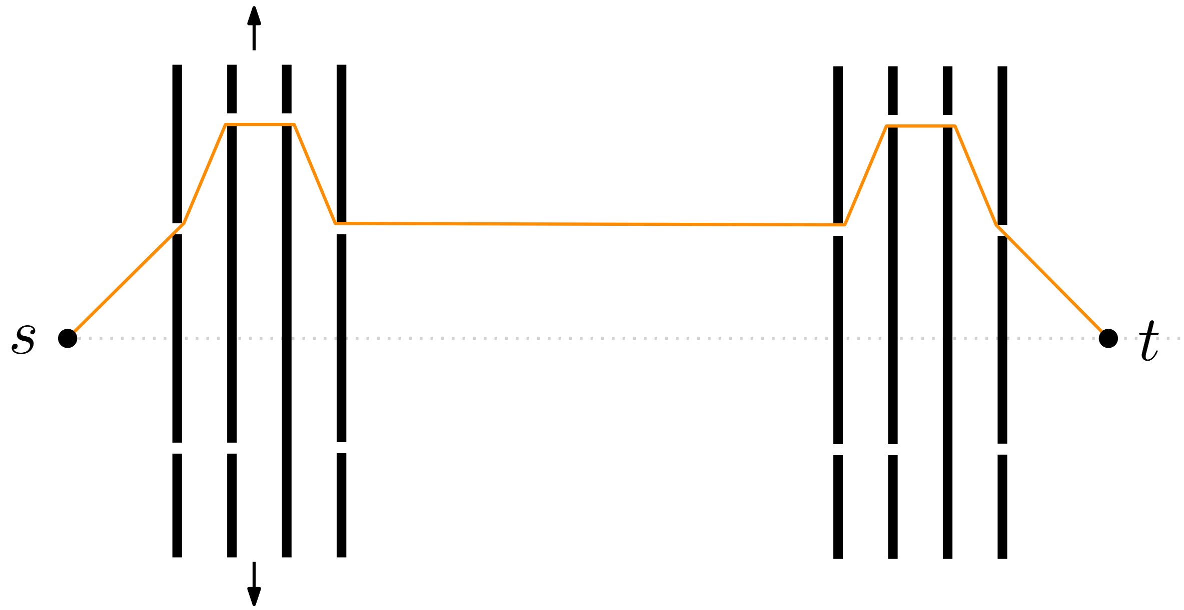

- Shortest 0-path: $D + 2\sqrt{2} + 2\delta$

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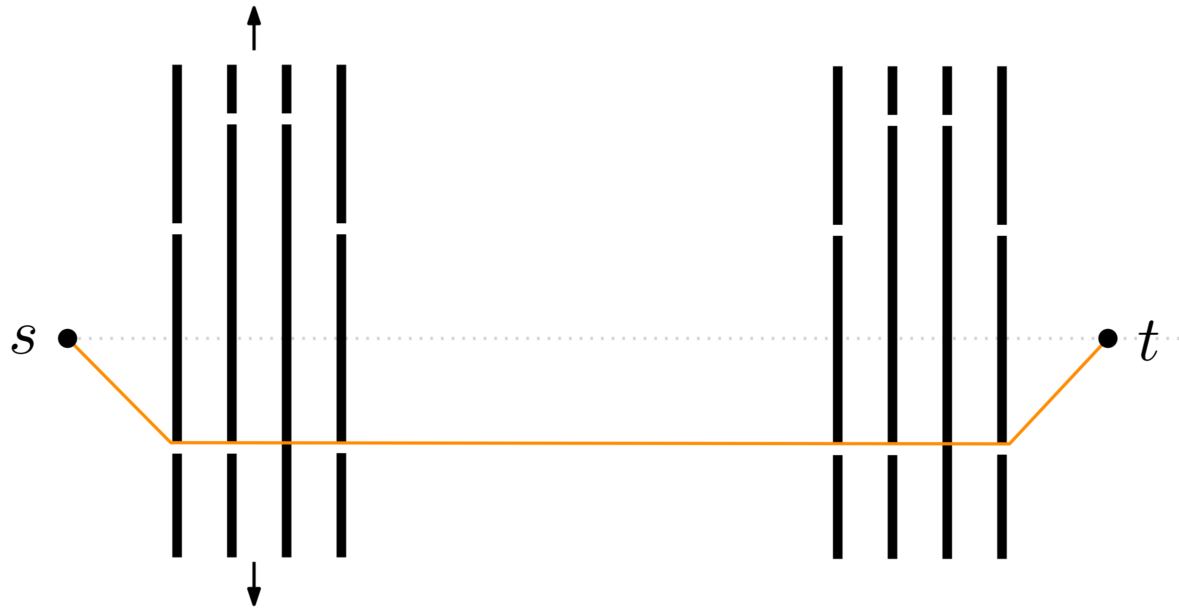

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➨ Shortest 2-path: $D + 2\sqrt{2}$

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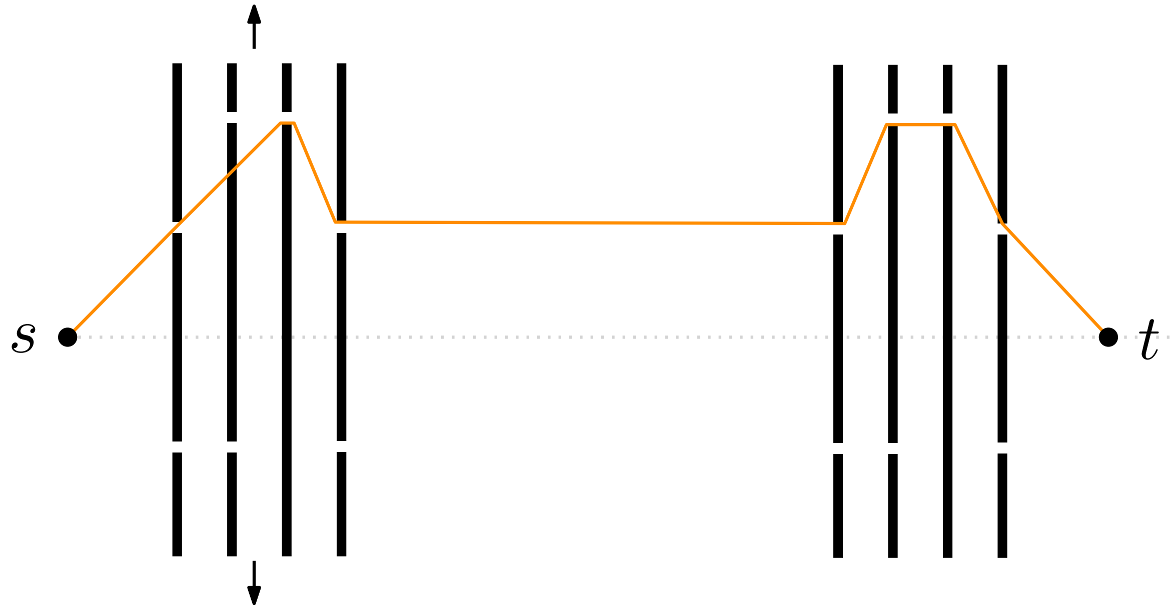

Shortest 0-path: $D + 2\sqrt{2} + 2\delta$

Shortest 2-path: $D + 2\sqrt{2}$

➡ 1-path crossing obstacle in upper group $> D + 2\sqrt{2} + 3\delta/2$

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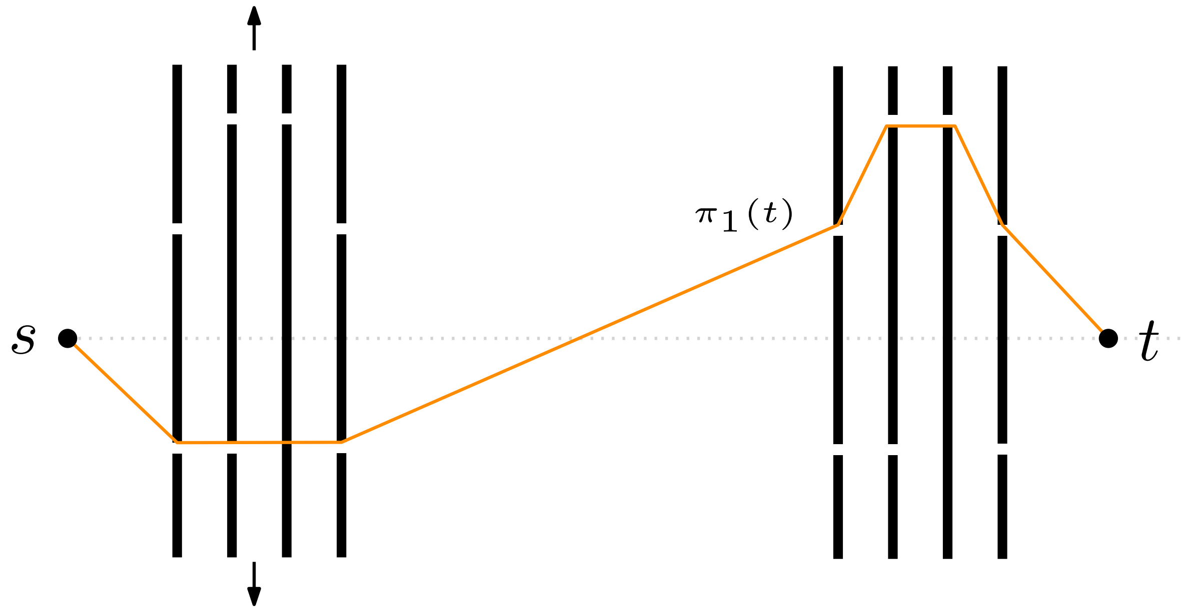

Setting $D = 10$, $\delta = 0.4$ makes this path shortest 1-path

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Shortest 2-path: $D + 2\sqrt{2}$

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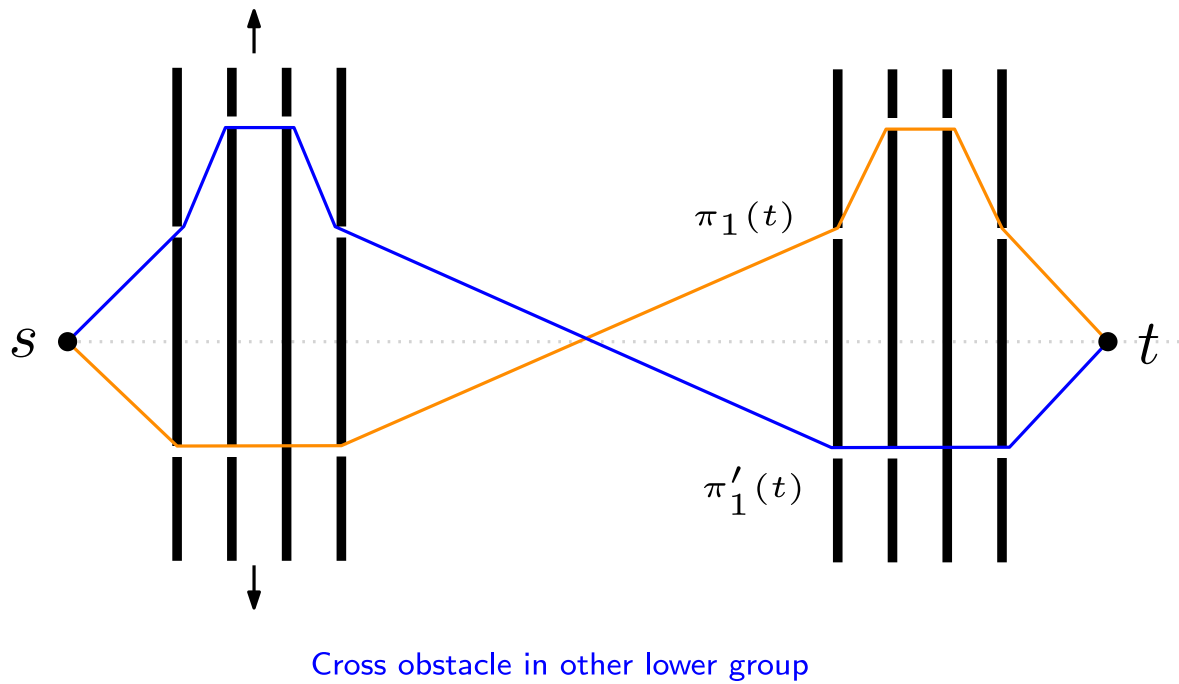

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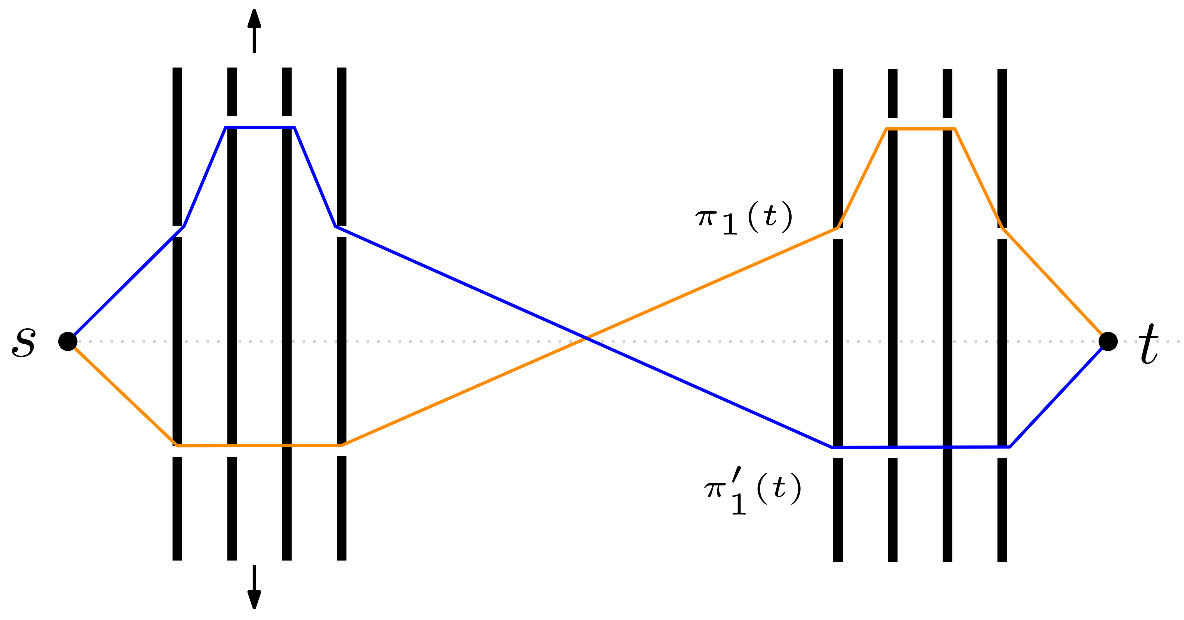

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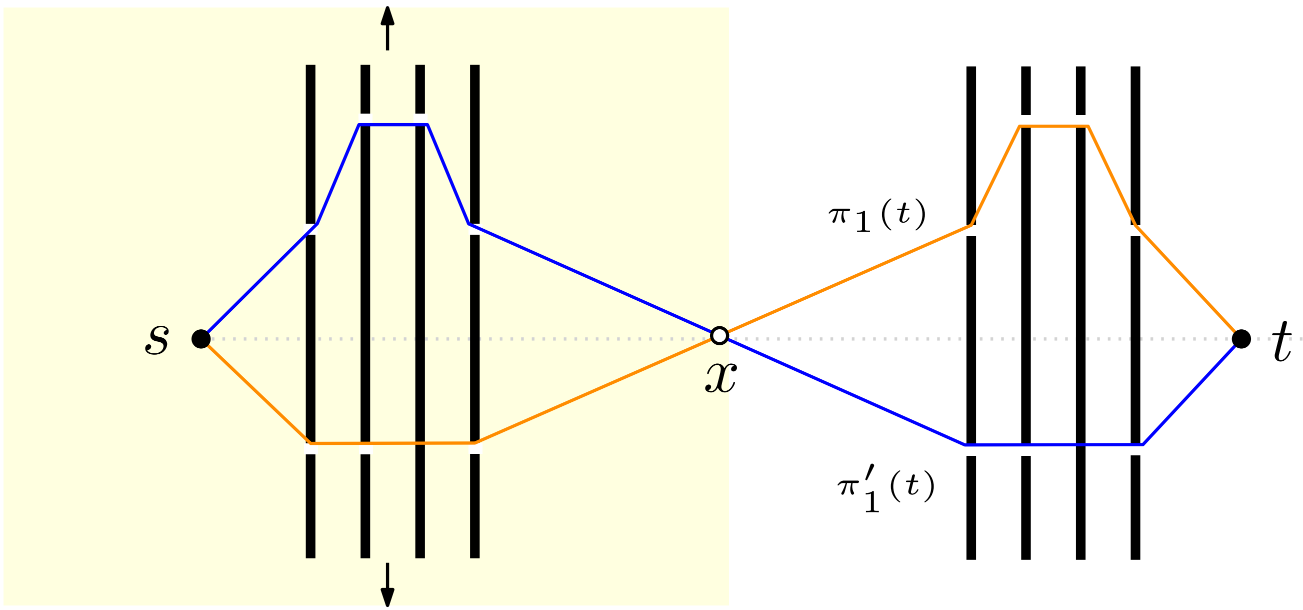

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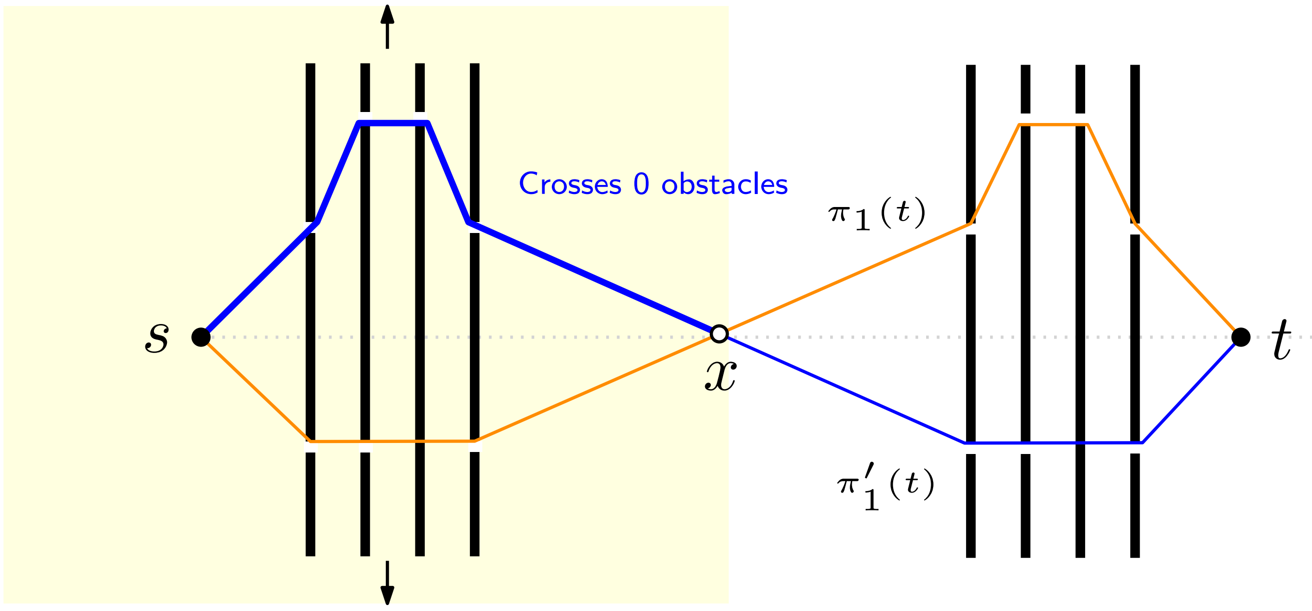

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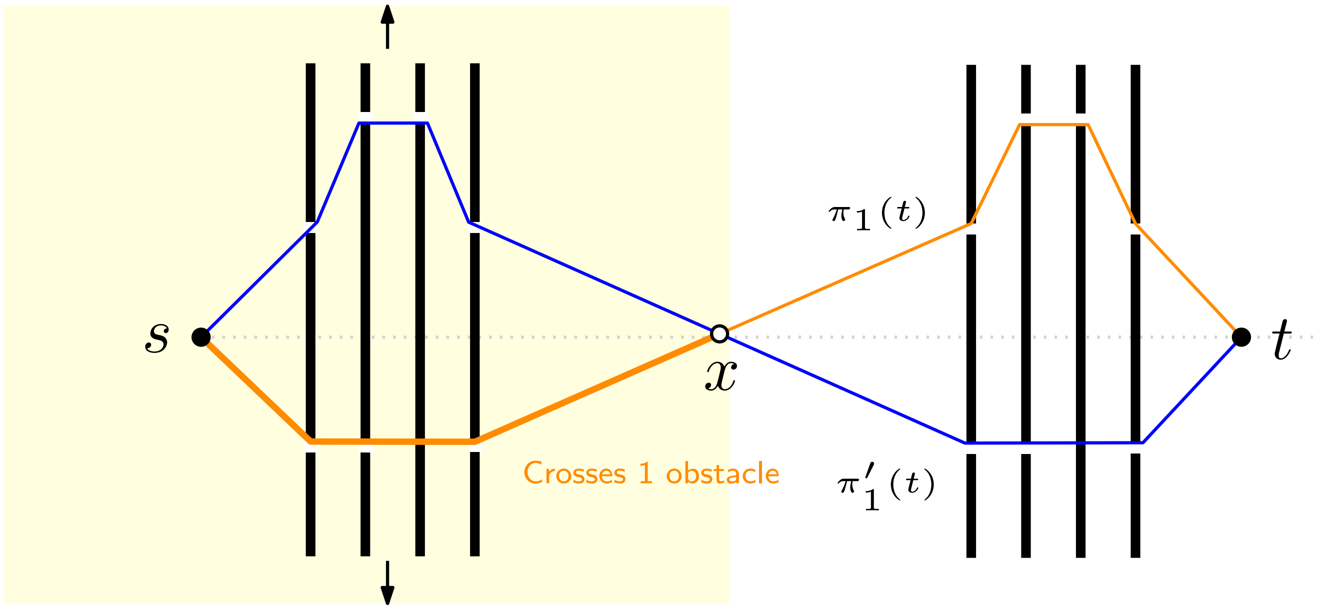
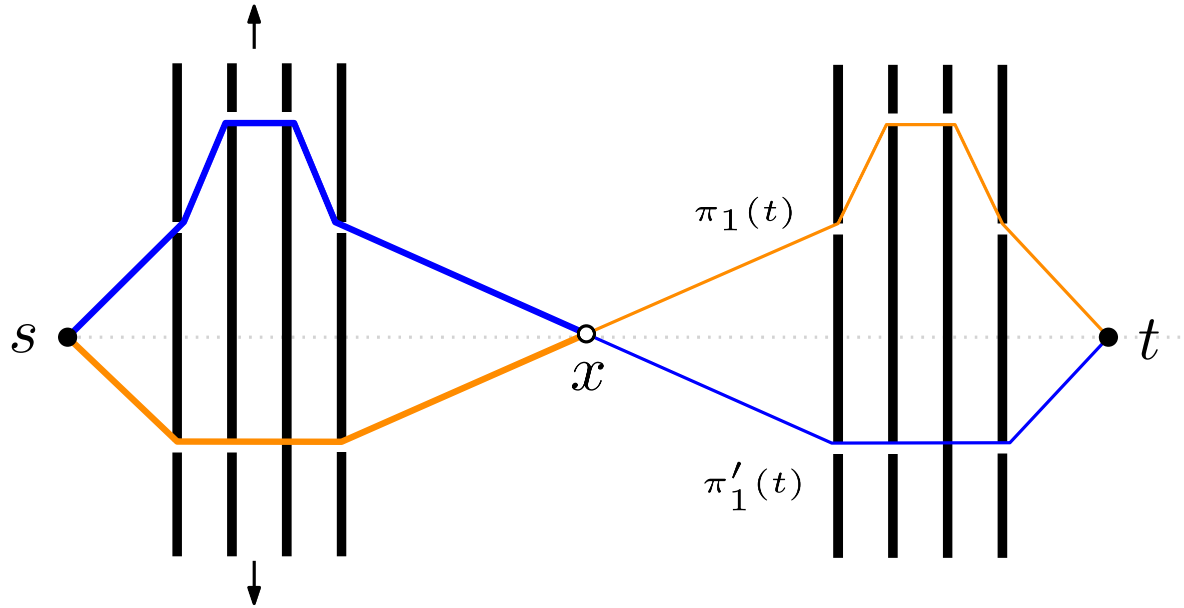


Observation

Prefix of crossing k-paths cross different number of obstacles

Shortest k-Paths may Cross!



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k-Paths as Non-crossing Subpaths

Prefix count of a k-path π_k at some point p on π_k

$\equiv$ Number of obstacles crossed by subpath of π_k until p

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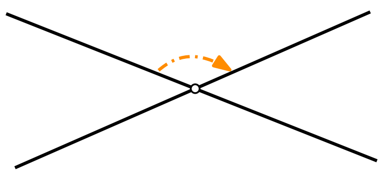

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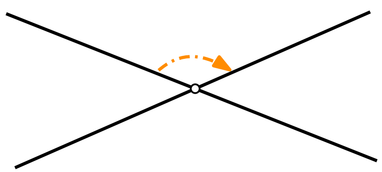

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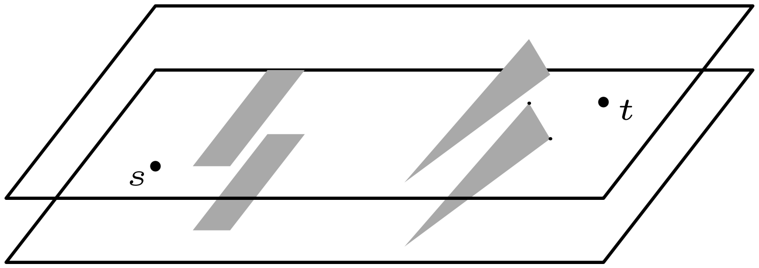
Subpaths with same prefix count do not cross!

Use this to 'separate' relevant k-paths with different prefix counts and apply Continuous Dijkstra.

k-Level Garage with Elevators

– Stack $(k+1)$ copies of P



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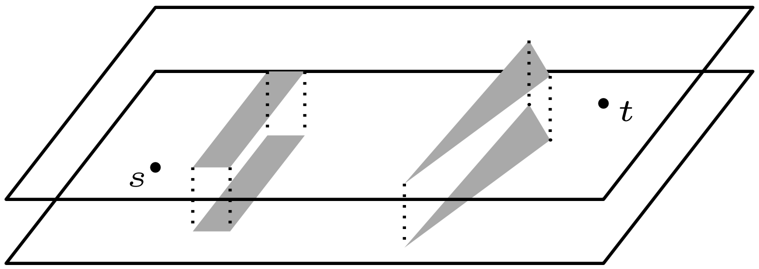

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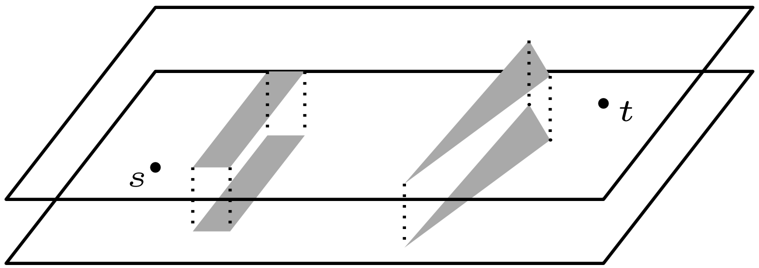

– Paths ascend floors by going inside obstacles (elevators)

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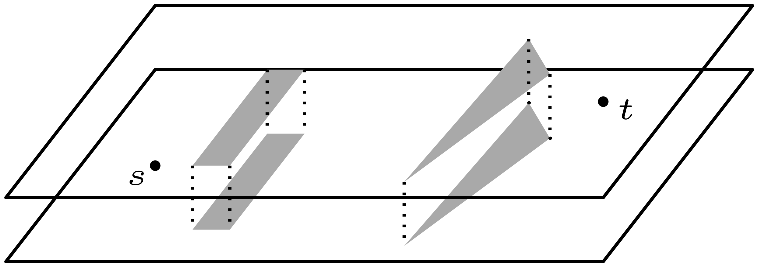

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– Floors correspond to prefix counts

All paths reaching floor i have the prefix count $i \Rightarrow$ non-crossing

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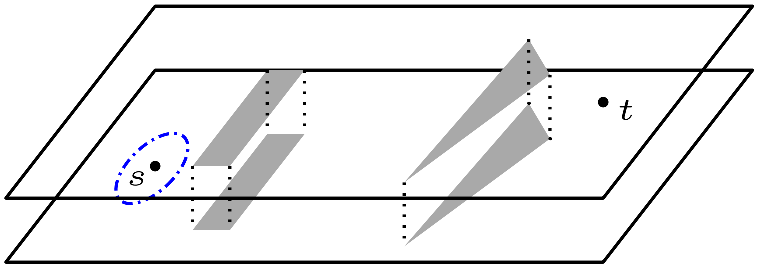

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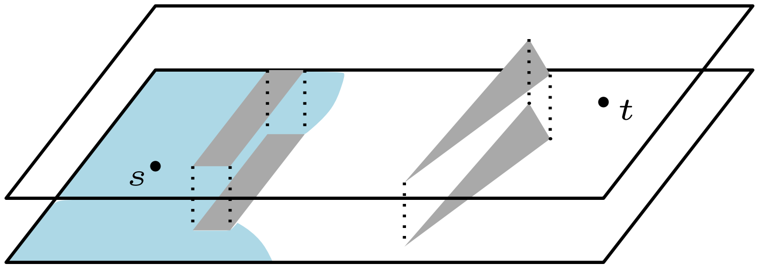

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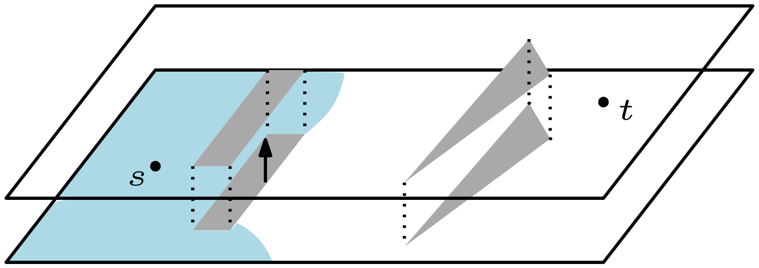

Wavelets move up floors by entering obstacles

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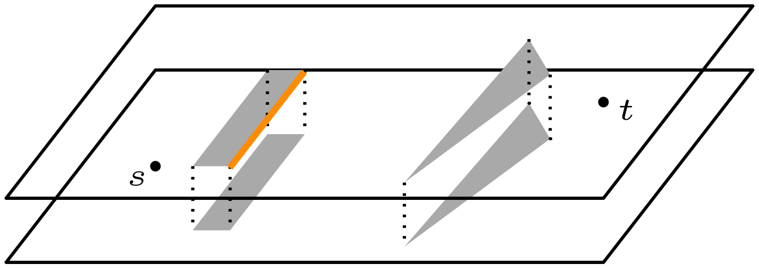

New 'sources' added at next level

– Paths ascend floors by going inside obstacles (elevators)

– Floors correspond to prefix counts

All paths reaching floor i have the prefix count i $\Rightarrow$ non-crossing

Propagate wavefronts across floors starting at s

k-Level Garage with Elevators

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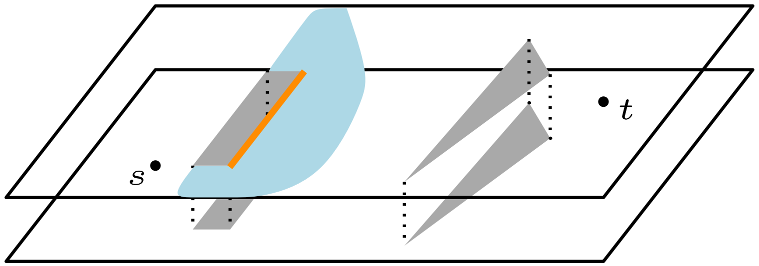

Continue wavefront propagation as usual

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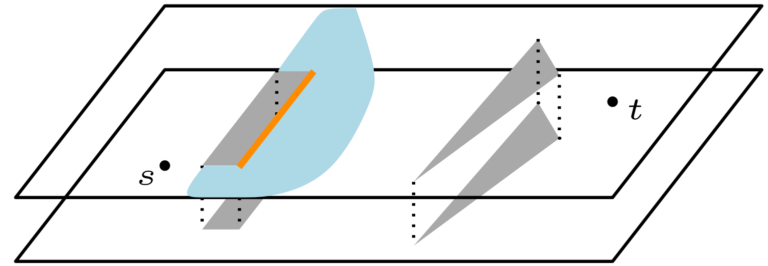

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Propagate wavefronts across floors starting at s

– Planar subdivision at floor k is shortest $(=k)$-path map $SPM_{=k}$.

k-Level Garage with Elevators

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For wavefront propagation at floor i, sources from floor $i-1$ suffice

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Why?

k-Level Garage with Elevators

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Why? Path Decomposition

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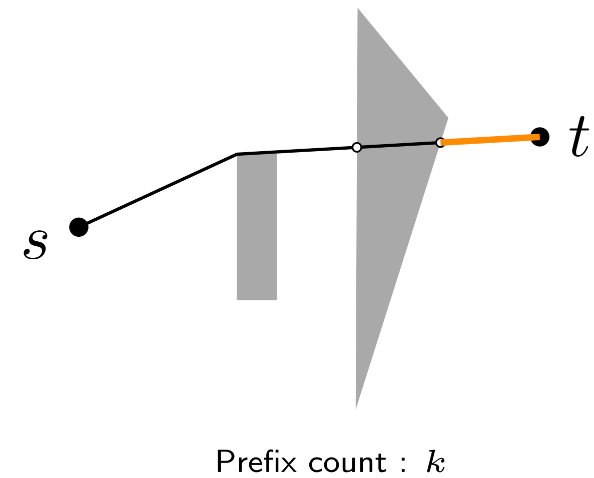
Shortest k-path can be decomposed into

– Shortest 0-Path



k-Level Garage with Elevators

For wavefront propagation at floor i, sources from floor $i-1$ suffice

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Shortest k-path can be decomposed into

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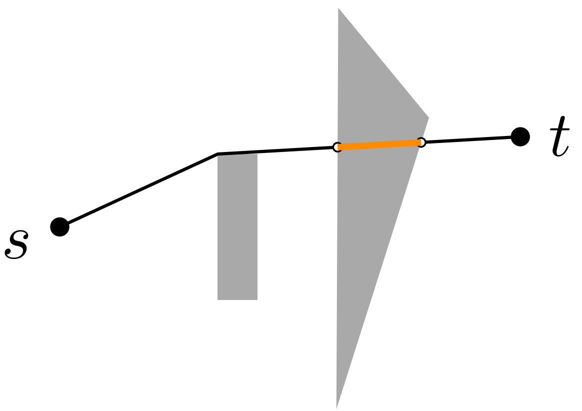


Prefix count : $(k - 1)$

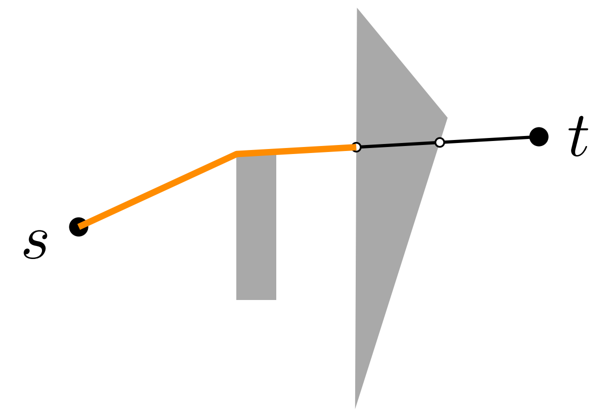
k-Level Garage with Elevators

For wavefront propagation at floor i, sources from floor $i-1$ suffice

Why? Path Decomposition

Shortest k-path can be decomposed into

- Shortest 0-Path
- Segment inside an obstacle
- Shortest $(k-1)$-path



Decompose recursively into $2k-1$ disjoint subpaths

Computing Shortest k-path map (SPM_k)

Partition of *free space* into regions with same k-predecessor

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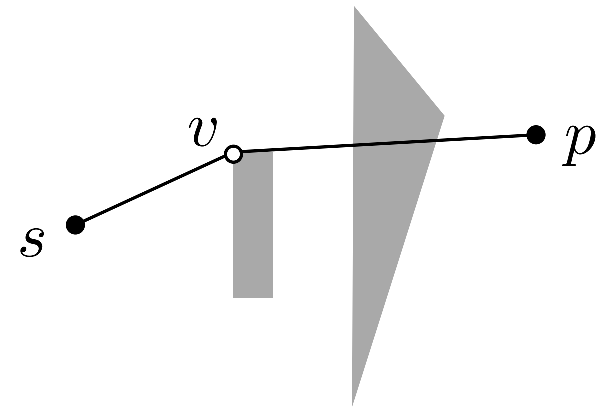
– Identify k-predecessor of p as (v, i)

v is adjacent to p on $\pi_k(p)$ and $\overline{vp}$ crosses $(k - i)$ obstacles



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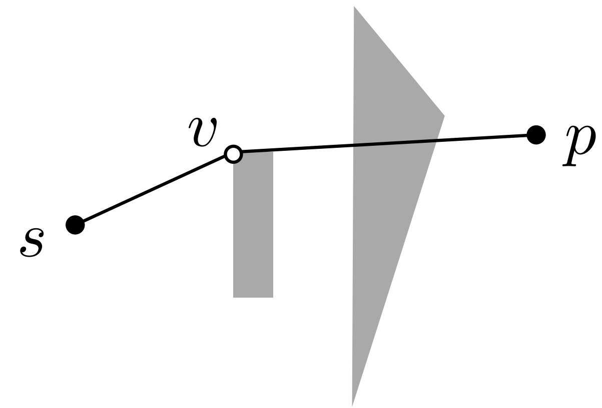

$(v\,,\,0)$ is 1-predecessor of p

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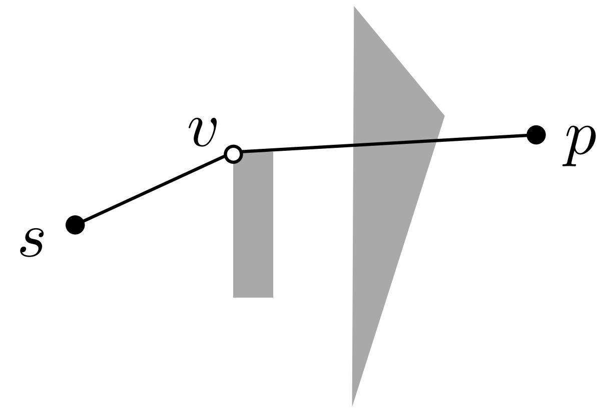

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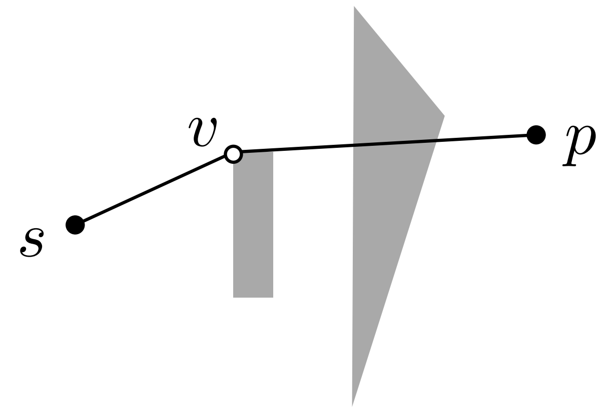
Total $O(kn)$ predecessors



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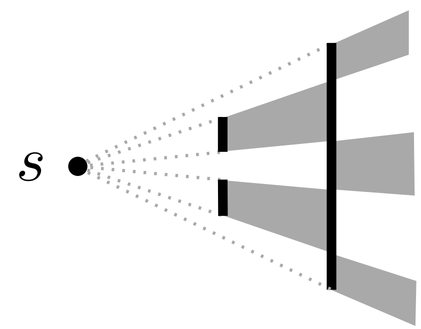

Multiple regions can have the same k-predecessor

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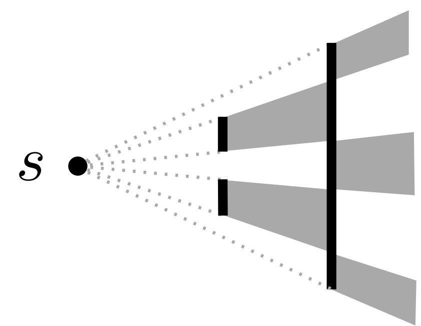

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– Comprises of two distinct regions

V_{k-1} region visible from s by crossing fewer than k obstacles

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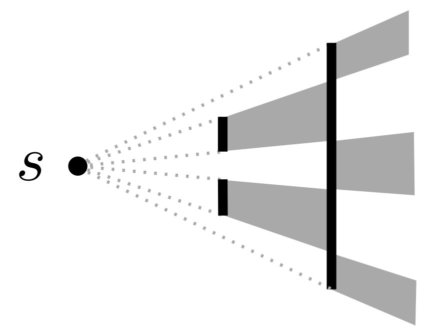

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$SPM_{=k}$ Rest of free space

Computing SPM_k

Computes both V_{k-1} and $SPM_{=k}$ one level at a time

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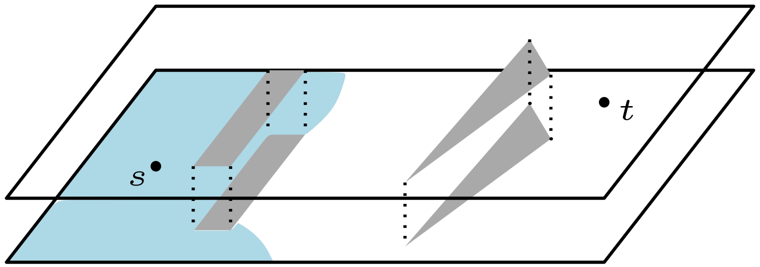
Propagate wavefront

$\Rightarrow$ At each garage floor



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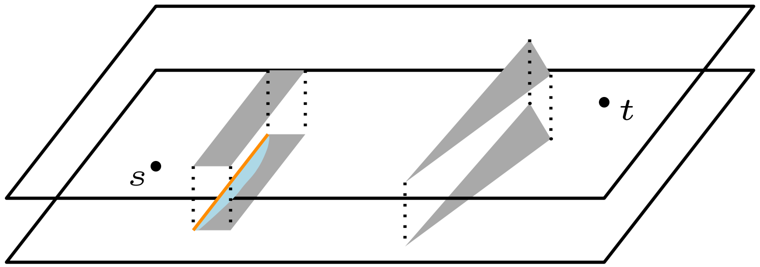
$\Rightarrow$ At each garage floor

$\Rightarrow$ Inside the obstacles (elavators)



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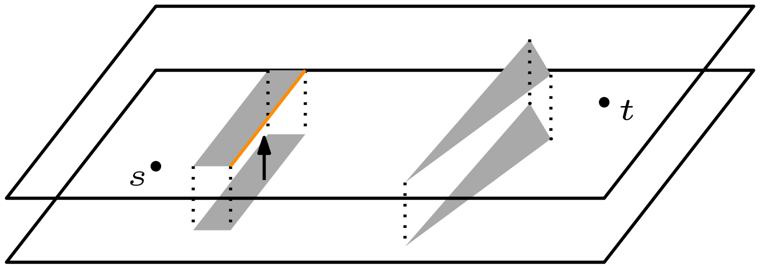

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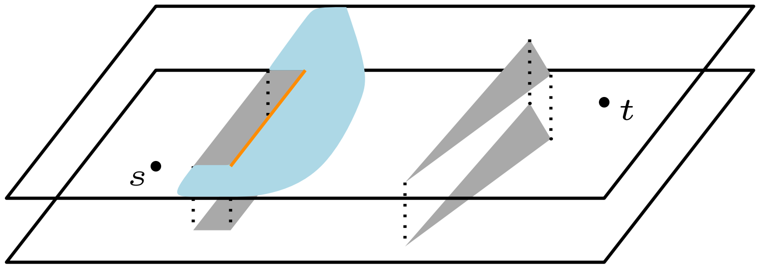
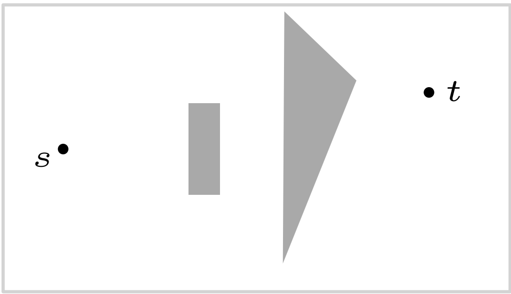


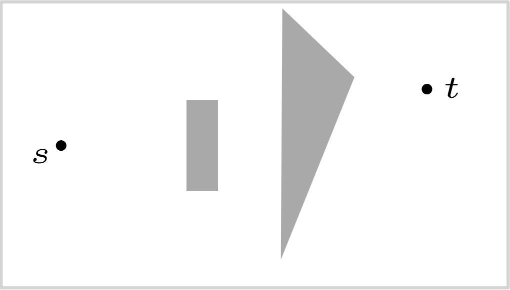
Computing SPM_k : Algorithm Outline



Computing SPM_k : Algorithm Outline

1. Set $\underline{M = \{s\}}$ and $V = \emptyset$.

Sources passed to HS-algorithm



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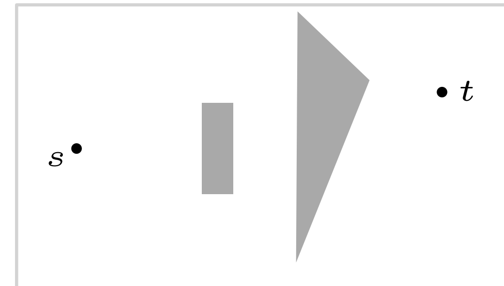
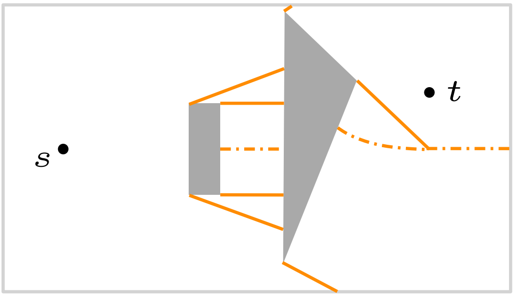
Visibility region V_{i-1}



Computing SPM_k : Algorithm Outline

1. Set $M = \{s\}$ and $V = \emptyset$.

   Call HS-algorithm to compute SPM_0



Sources claim intervals on domain boundary

Computing SPM_k : Algorithm Outline

1. Set $M = \{s\}$ and $V = \emptyset$.

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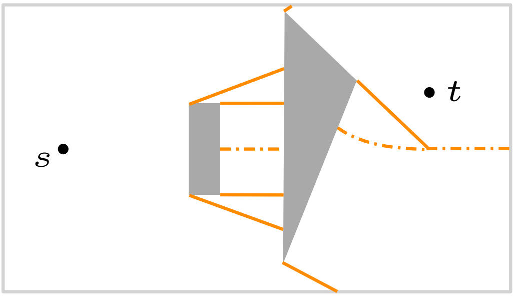
2. For $i \in 1, 2, \ldots, k$



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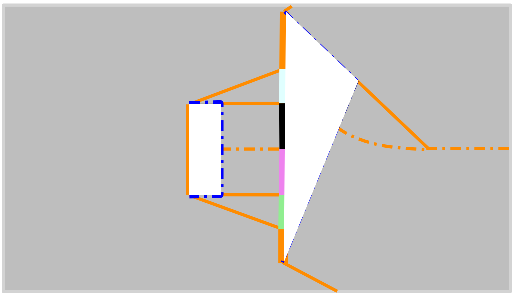
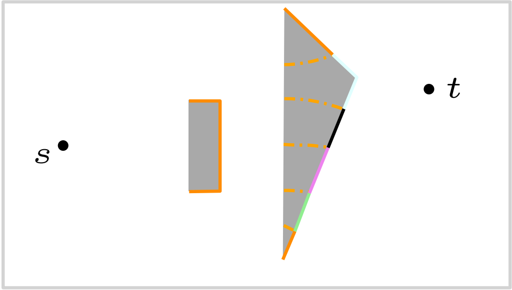


'Boundary sources' for propagation within obstacles

2. For $i \in 1, 2, \ldots, k$

   $\Rightarrow$ Propagate 'claims' in SPM_{i-1} inside the obstacles

Computing SPM_k : Algorithm Outline



Boundary sources for next level

1. Set $M = \{s\}$ and $V = \emptyset$.

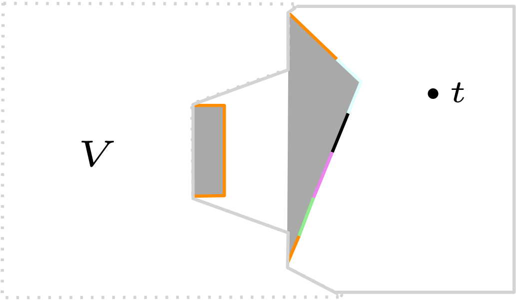
   Call HS-algorithm to compute SPM_0

2. For $i \in 1, 2, \ldots, k$

   $\Rightarrow$ Propagate 'claims' in SPM_{i-1} inside the obstacles

   Gives new boundary sources say M_{new}

Computing SPM_k : Algorithm Outline



$P = P - V$

1. Set $M = \{s\}$ and $V = \emptyset$.

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2. For $i \in 1, 2, \ldots, k$

   $\Rightarrow$ Propagate 'claims' in SPM_{i-1} inside the obstacles

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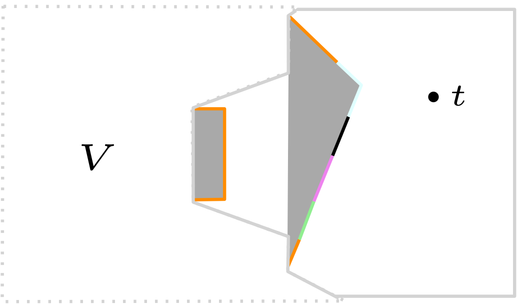
   $\Rightarrow$ Drop regions of SPM_{i-1} with s as predecessor from P

   k-visible from s, include this region to V.

Computing SPM_k : Algorithm Outline



Continue propagation with new sources

1. Set $M = \{s\}$ and $V = \emptyset$.

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2. For $i \in 1, 2, \ldots, k$

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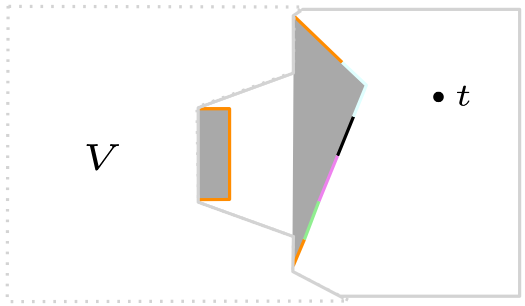
   $\Rightarrow$ With M as M_{new}, call HS-algorithm on P to compute $SPM_{=i}$.

Computing SPM_k : Algorithm Outline



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   k-visible from s, include this region to V.

   $\Rightarrow$ With M as M_{new}, call HS-algorithm on P to compute $SPM_{=i}$.

3. At this point we have V as V_{k-1} and $SPM_{=k}$.

   Merge them to obtain SPM_k

Computing SPM_k : Running Time

If S_i is the size of SPM_i

$$\text{Total Running time} \quad = \quad \sum_{i\,=\,1,\,2,\,..k} O(S_i \log S_i)$$

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Calls HS-Algorithm

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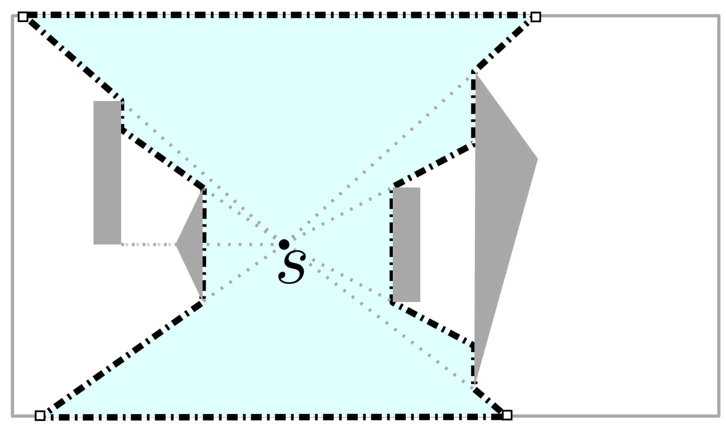
$$\leq O(k \cdot S_k \log S_k)$$

Compute an upper bound on size of SPM_k

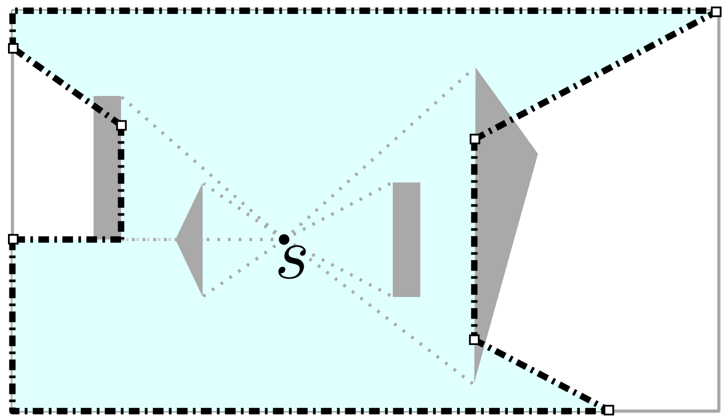
Bound sizes of V_{k-1} and $SPM_{=k}$

Complexity of k-visibility region V_k



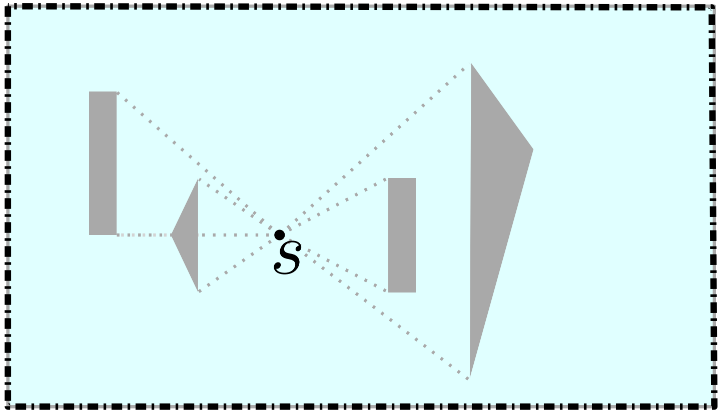
Region V_0

Complexity of k-visibility region V_k



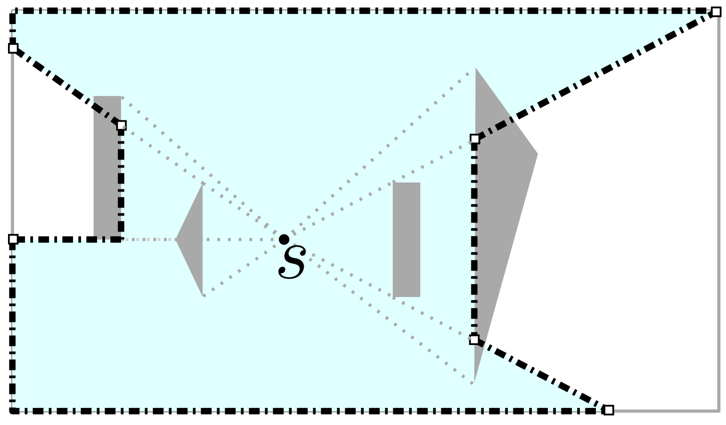
Region V_1

Complexity of k-visibility region V_k



Region V_2

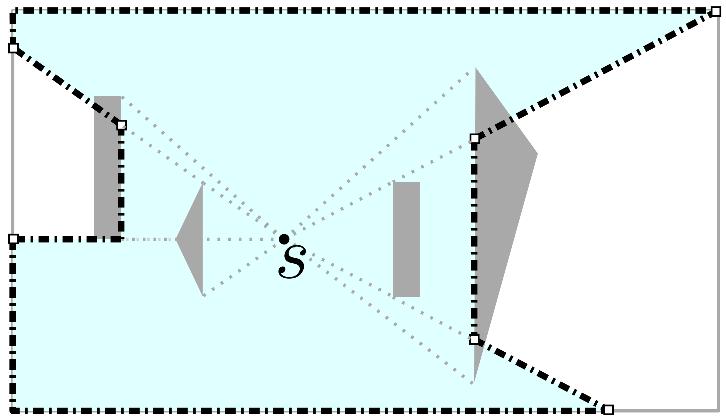
Complexity of k-visibility region V_k



The number of edges on boundary of V_k is $O(n+h)$

Vertex on ∂V_k is an obstacle vertex or projection of a tangent

Complexity of k-visibility region V_k



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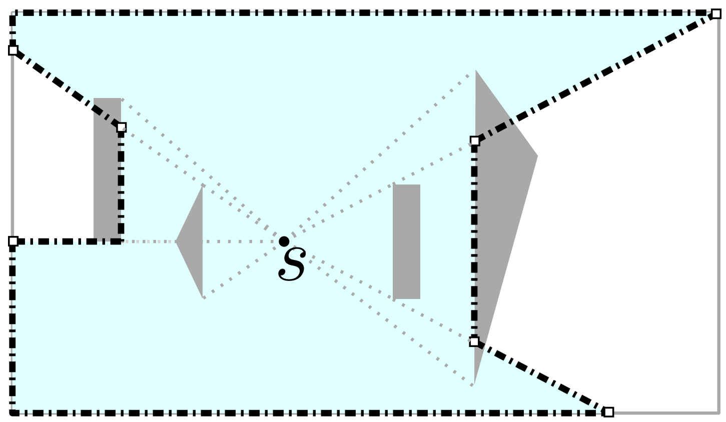

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Total complexity summed over all ∂V_i for $0 \leq i \leq k$ is $O(n+hk)$

Complexity of $SPM_{=k}$

Complexity of a map with m sources and n vertices is $O(m+n)$

Complexity of $SPM_{=k}$

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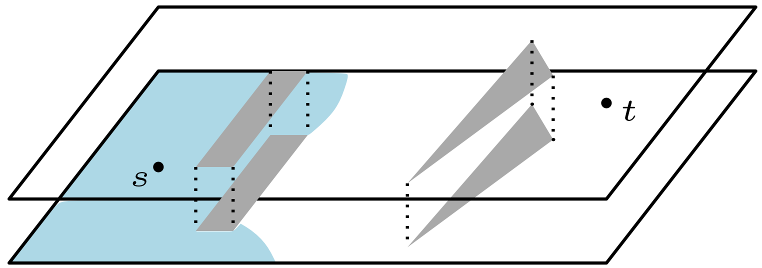
$SPM_{=k}$ may contain sources for wavelets from lower floor



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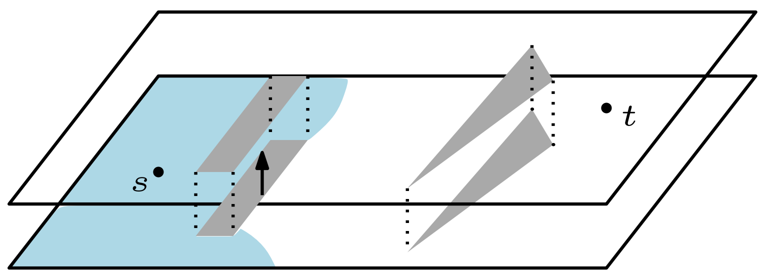

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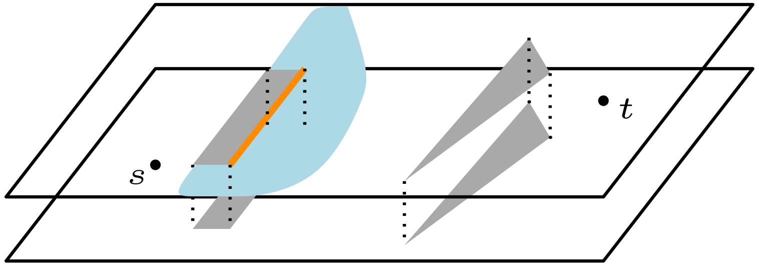

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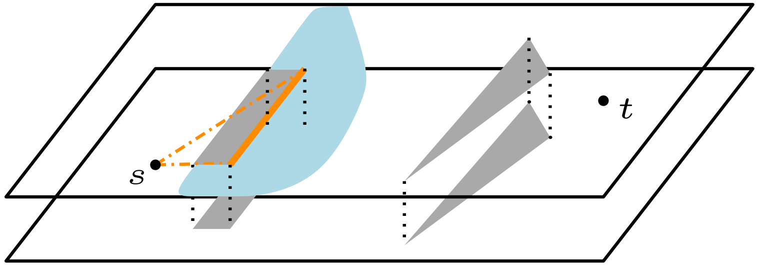

boundary sources : wavefront travel to obstacle boundary

by ascending one or more elevators

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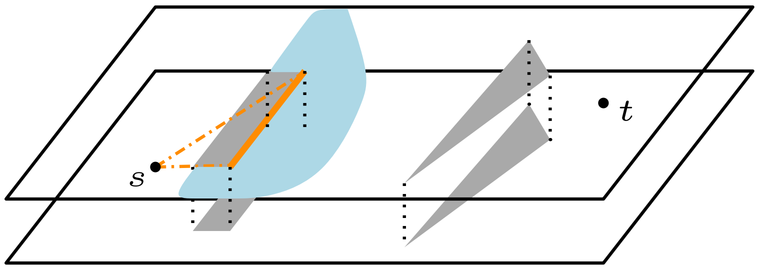
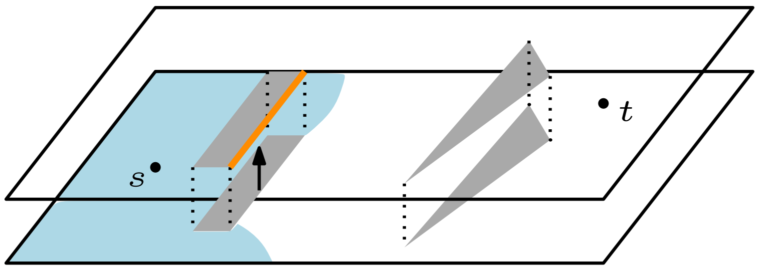


boundary sources : wavefront travel to obstacle boundary by ascending one or more elevators

Need to bound the number of such sources at each level

Complexity of $SPM_{=k}$: Boundary Sources

Boundary sources on level i are created by 'claims' at level $i-1$ when propagated inside the obstacle



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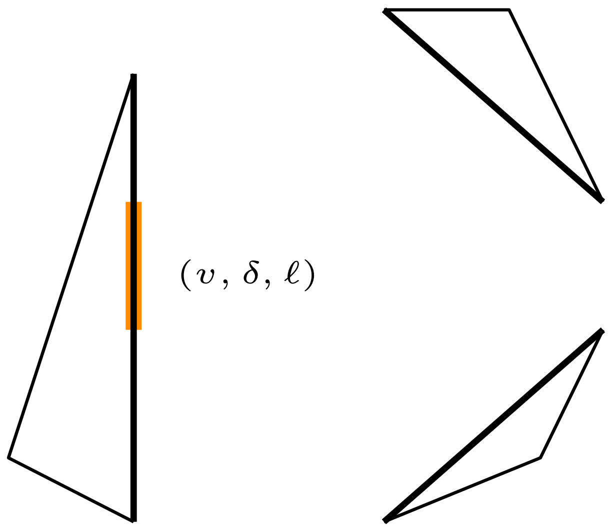
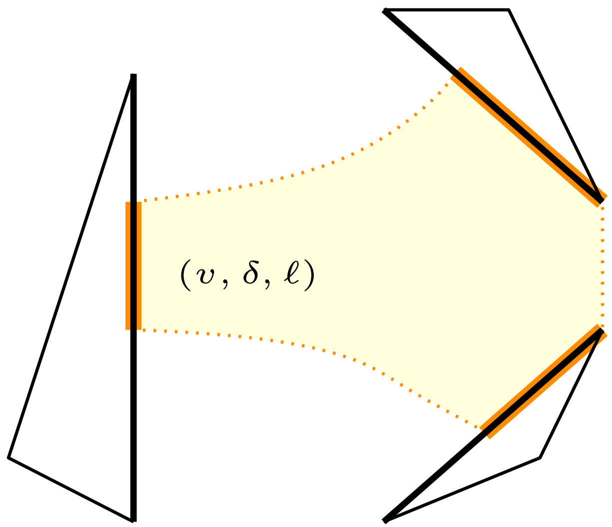


Two types of claims

– On the same edge : Entry Claim Cluster

Complexity of $SPM_{=k}$: Boundary Sources

Boundary sources on level i are created by 'claims' at level $i-1$ when propagated inside the obstacle



Two types of claims

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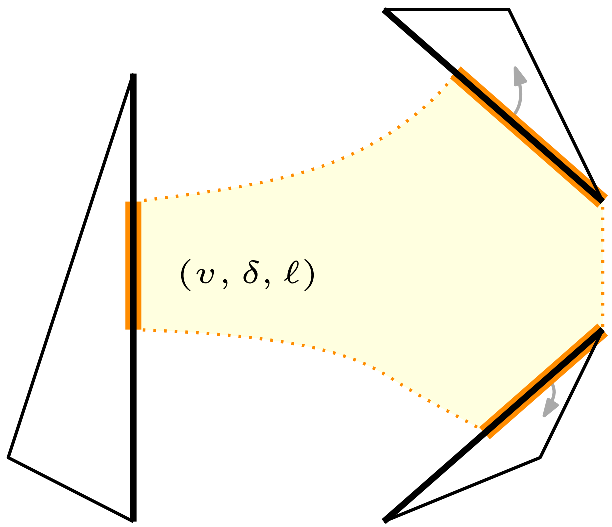

Two types of claims

- On the same edge : Entry Claim Cluster
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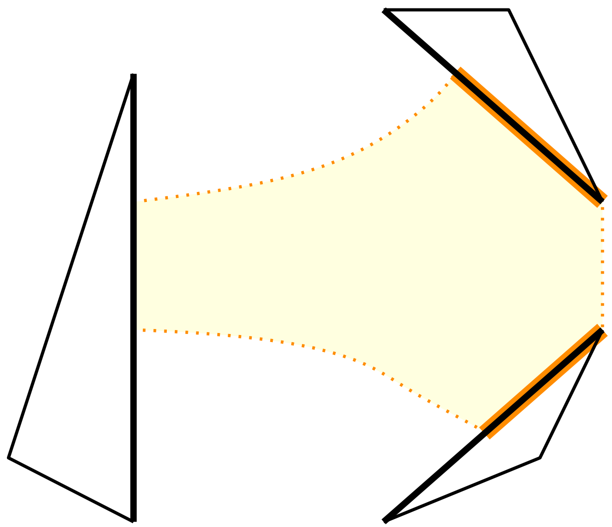
Only exit claims need to be propagated inside obstacles

Complexity of SPM_k

Lemma : The total number of exit claim clusters obtained by propagating m boundary sources in a domain is $m + O(n)$



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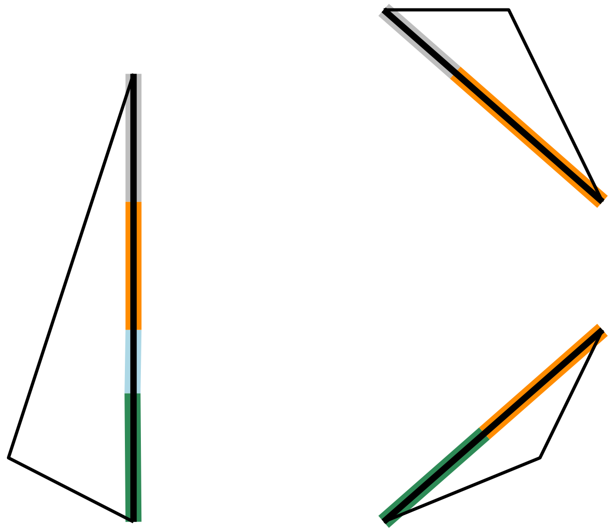
Proof sketch:

Connect exit claims of each of the m sources



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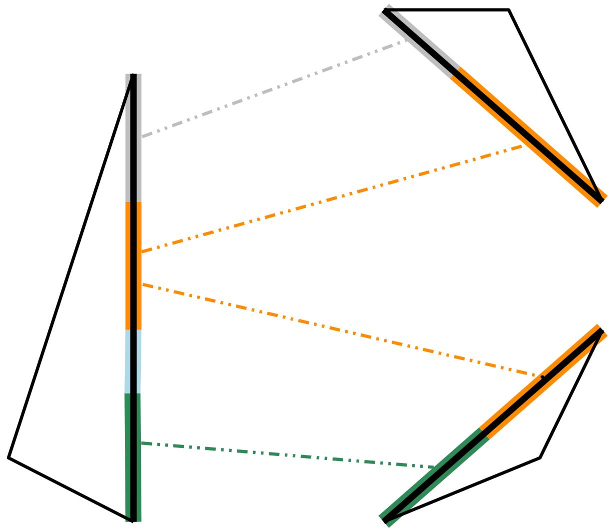

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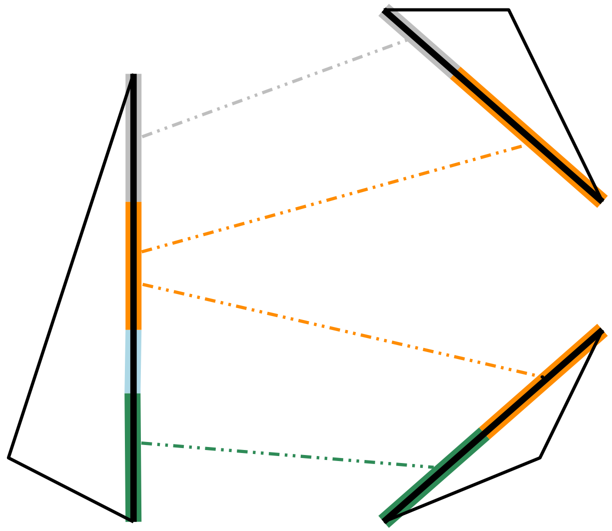

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With a planarity based 'technical' argument, we get the said bound

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Number of sources at level k

$$n + \underbrace{Cn + Cn + \ldots + Cn}$$

k times, one per level; C is some constant

Computing SPM_k : Running Time

If S_i is the size of SPM_i

$$\text{Total Running time} \quad = \quad \sum_{i=1,2,..k} O(S_i \log S_i)$$

$$\leq \quad O(k \cdot S_k \log S_k)$$

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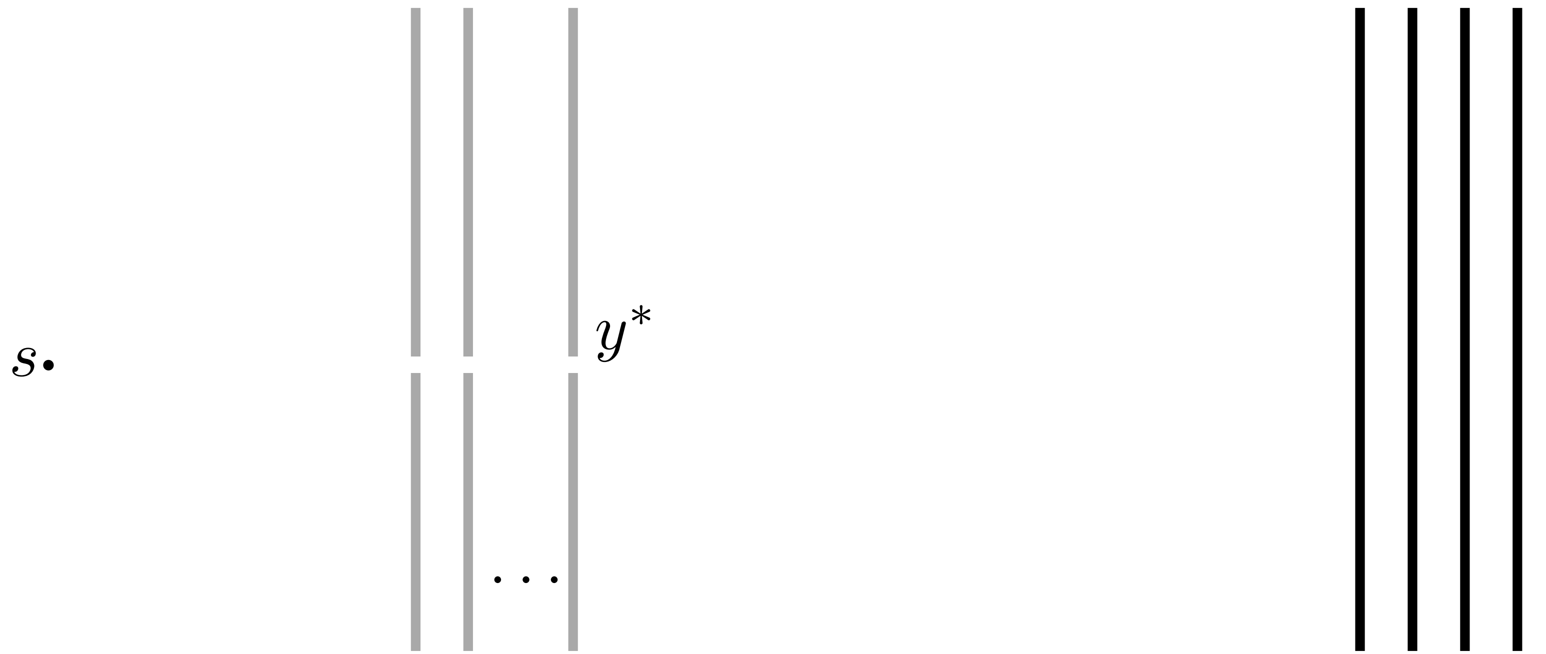
Theorem: SPM_k can be computed in $O(k^2 n \log n)$ total time and $O(kn \log n)$ space

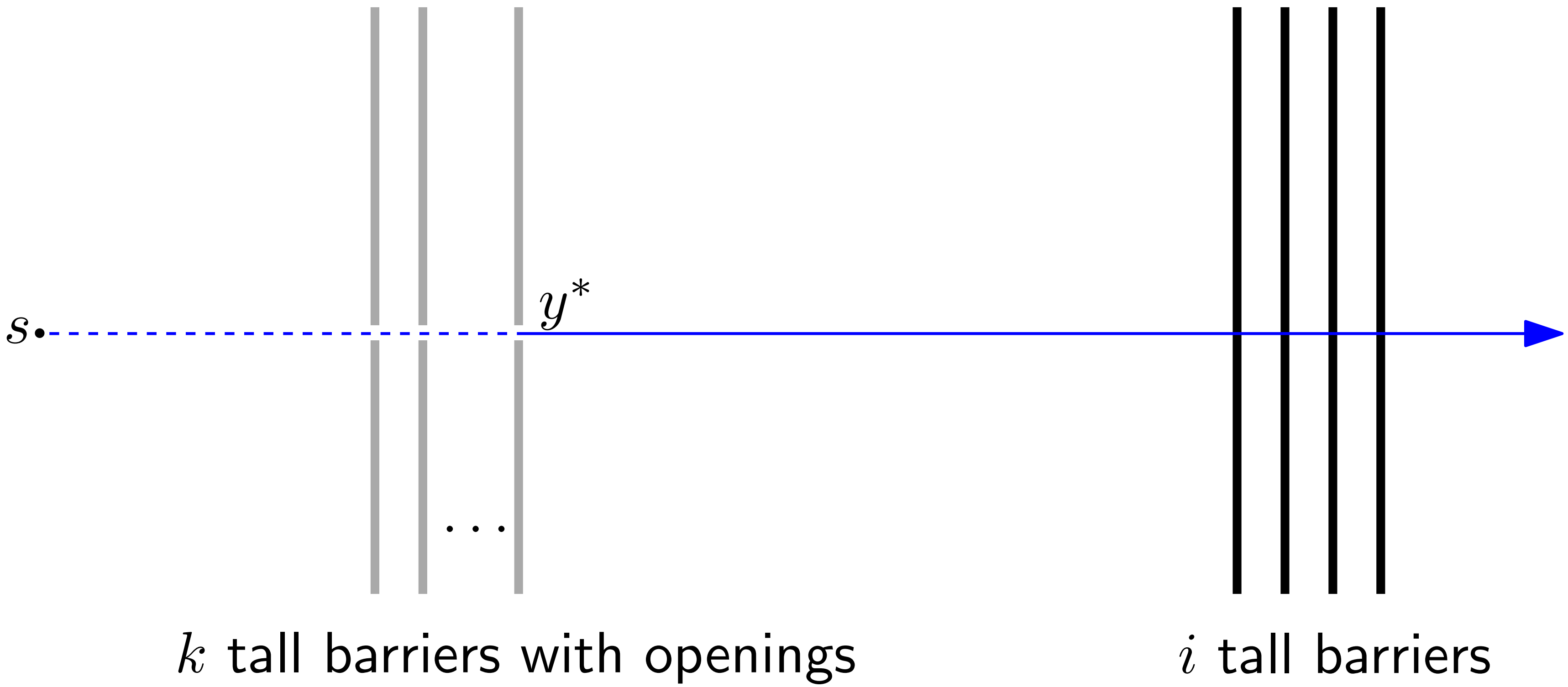
An $\Omega(kn)$ lower bound for SPM_k



k tall barriers with openings

i tall barriers

An $\Omega(kn)$ lower bound for SPM_k



Can have at most $k - i$ crossings before y^*

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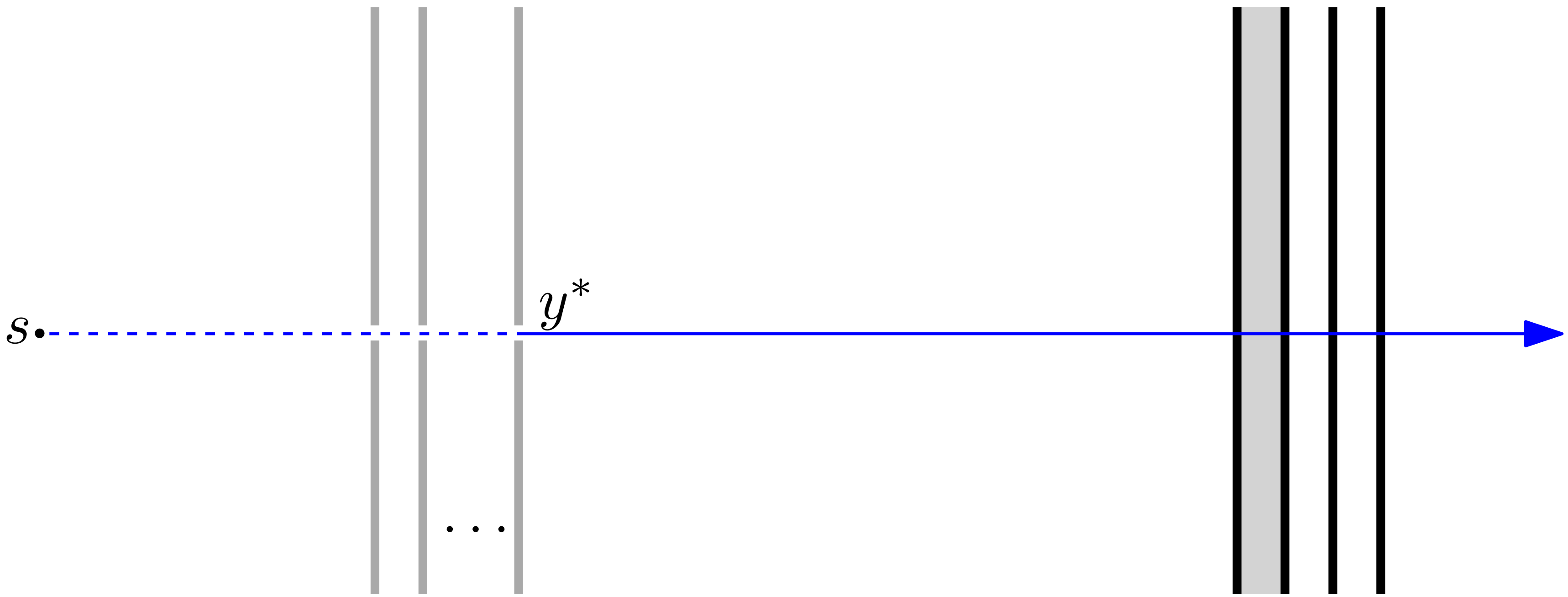

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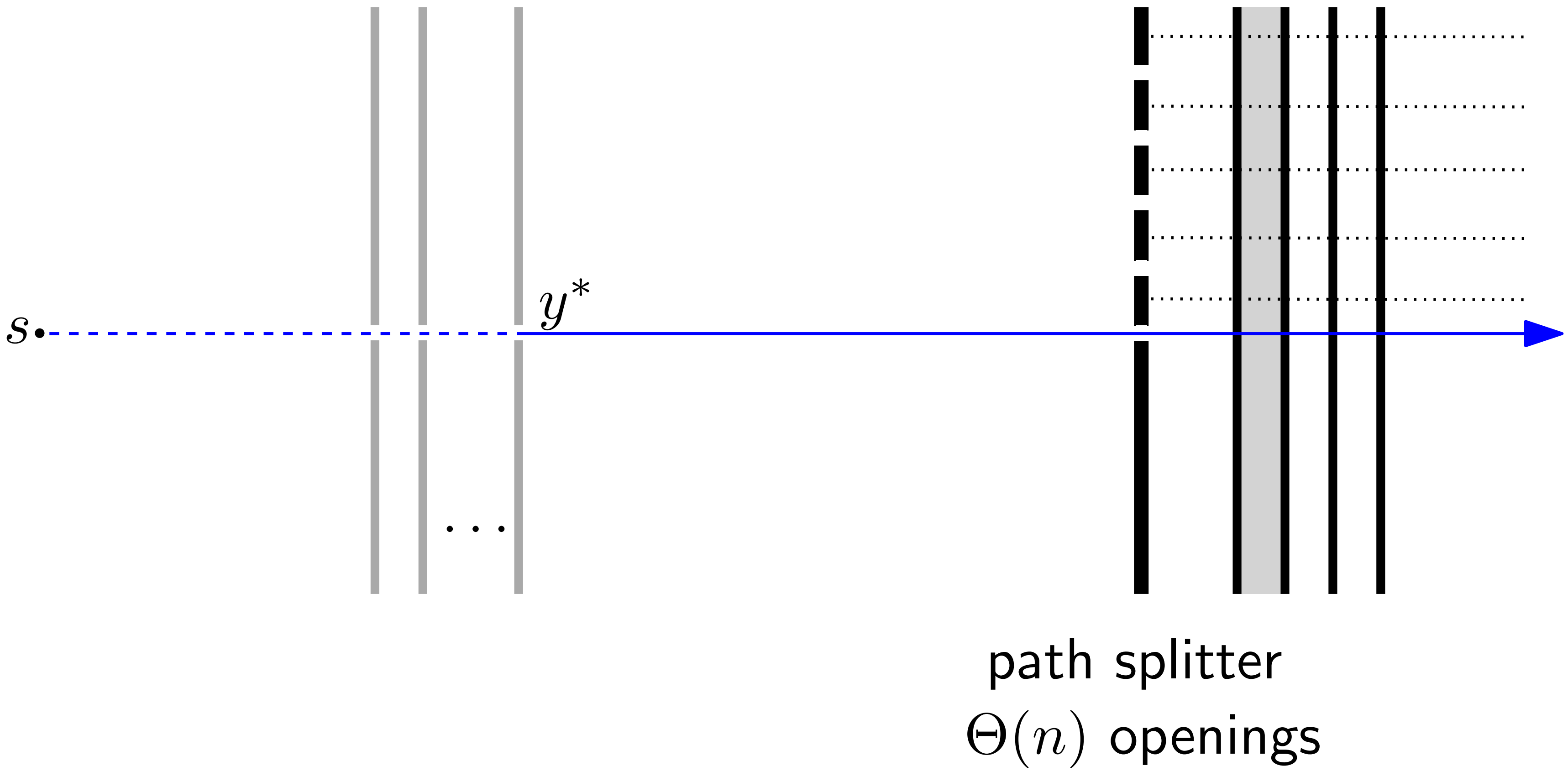

Can have at most $k - i$ crossings before y^*

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Creates $\Theta(n)$ sub-regions for every such region

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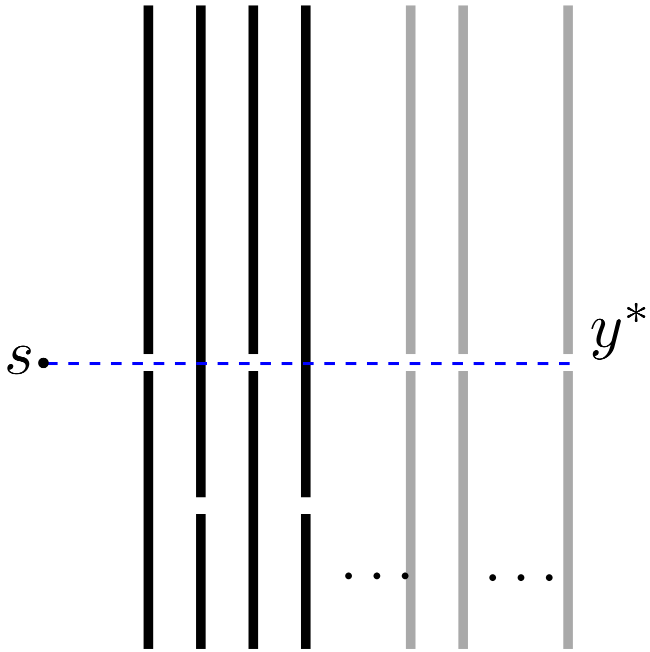
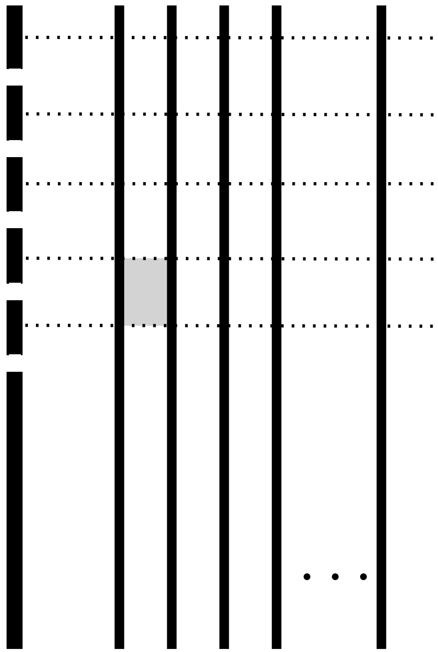


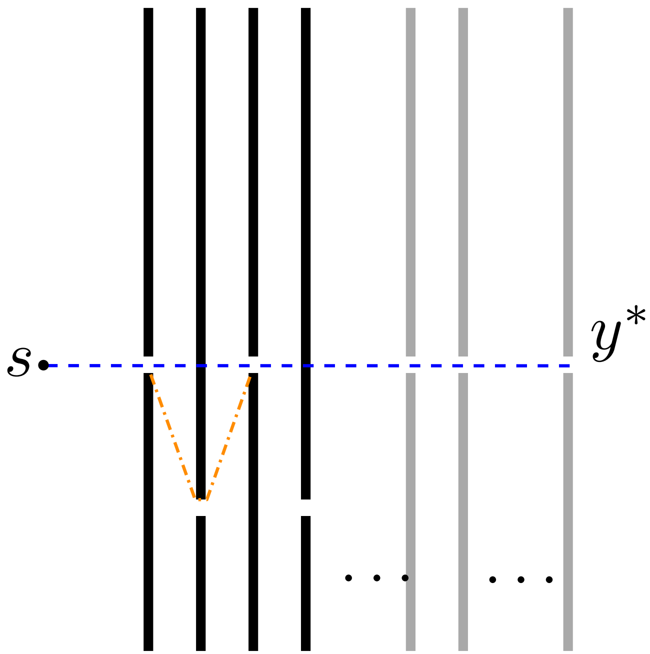
$2k$ strips



k strips

k-path from s to y^* can have 0 to k crossings

An $\Omega(kn)$ lower bound for SPM_k



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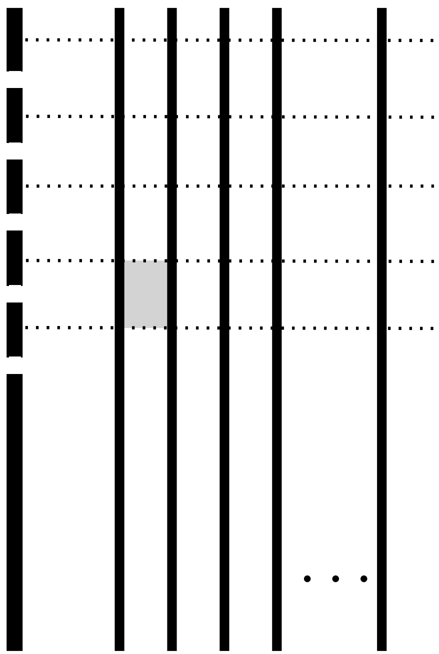

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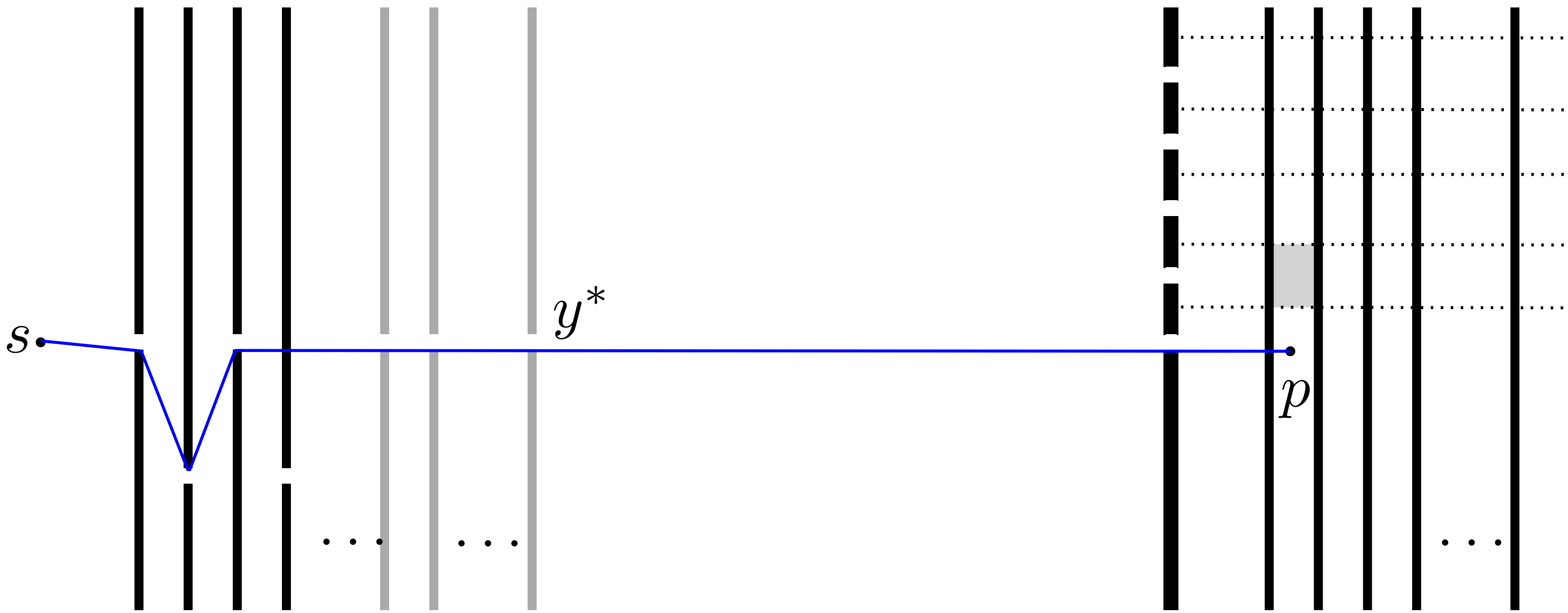

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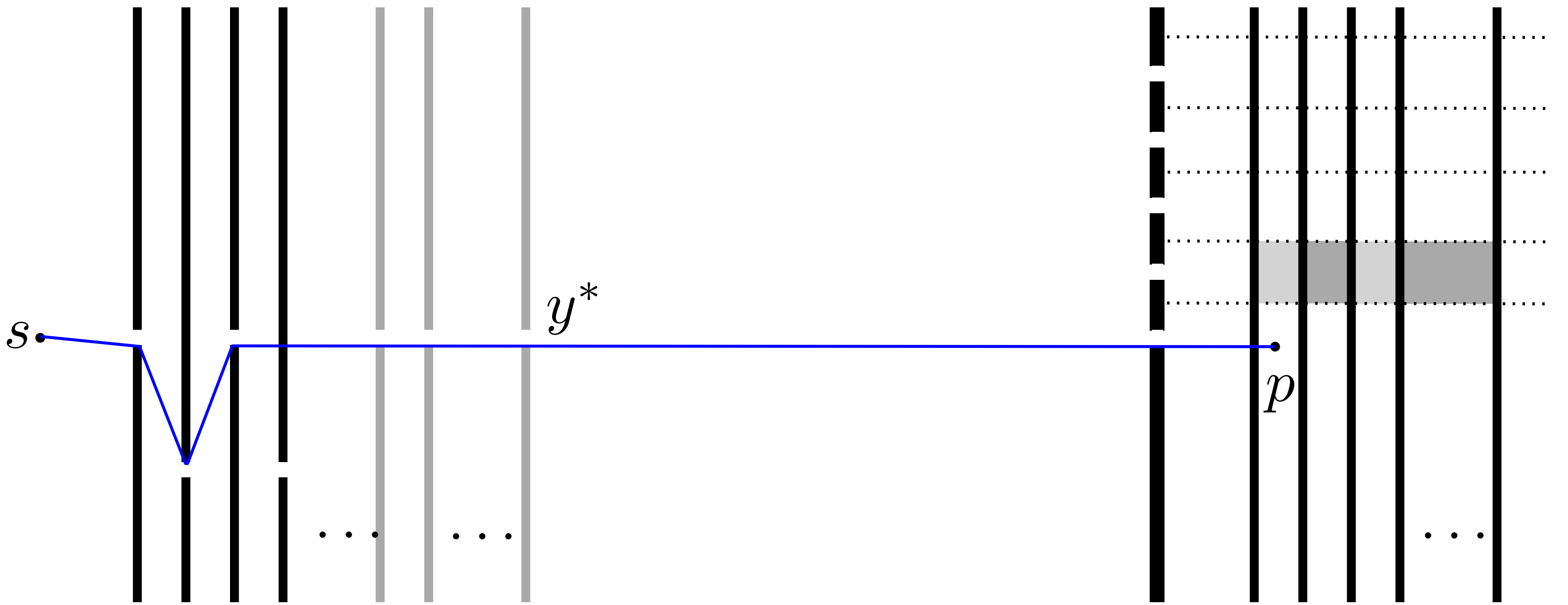

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Total $\Theta(k)$ regions per splitter opening, $\Theta(kn)$ regions in total.

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Thanks!