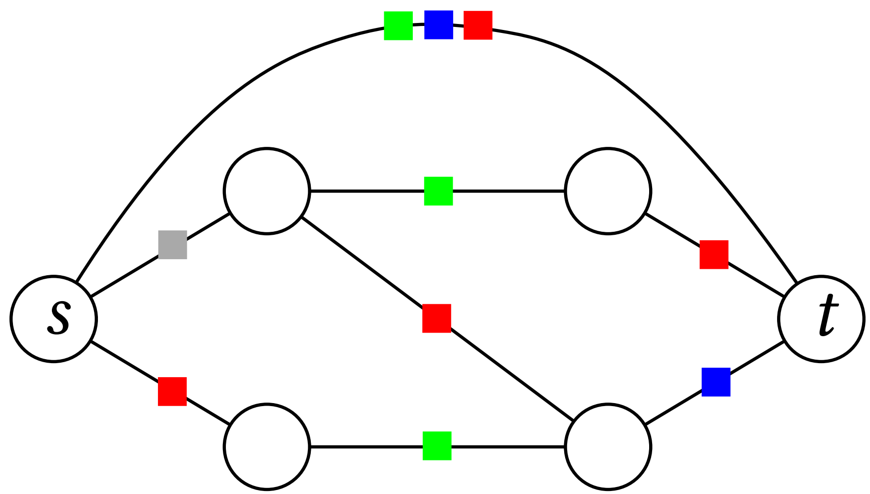
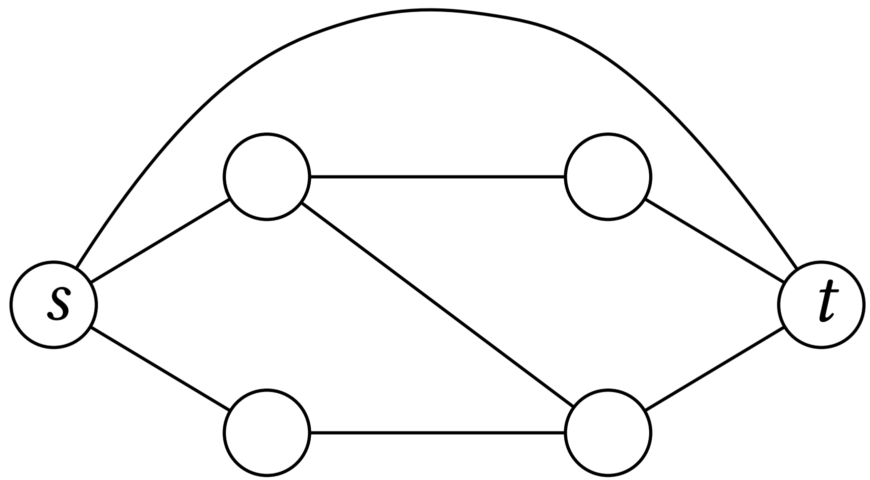


Computing a Minimum Color Path in Edge-Colored Graphs

Neeraj Kumar
University of California, Santa Barbara

Problem Description



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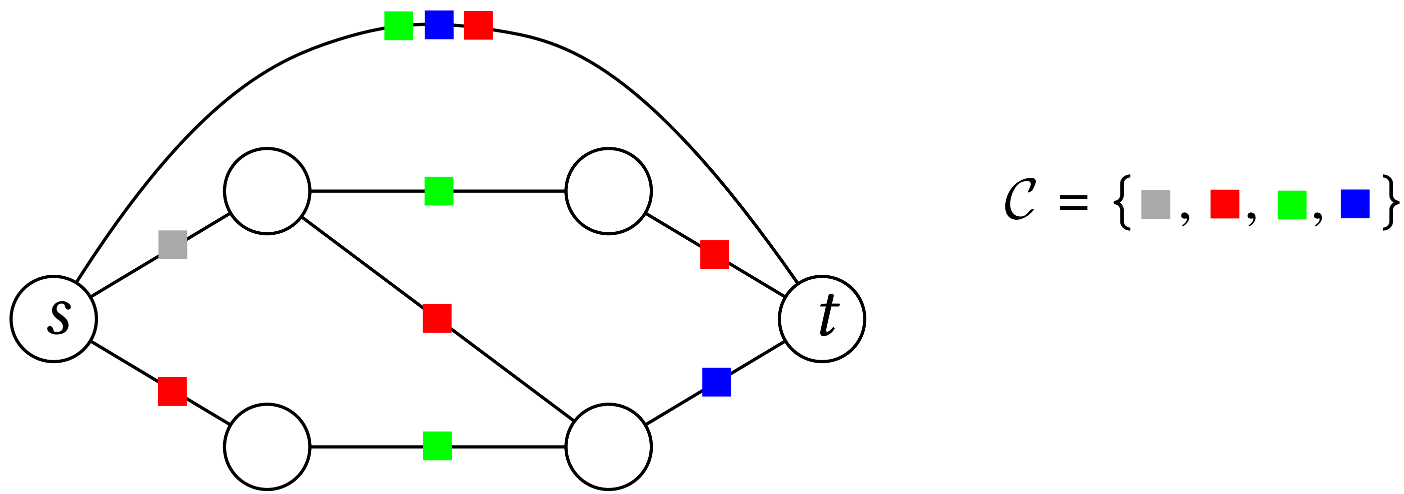

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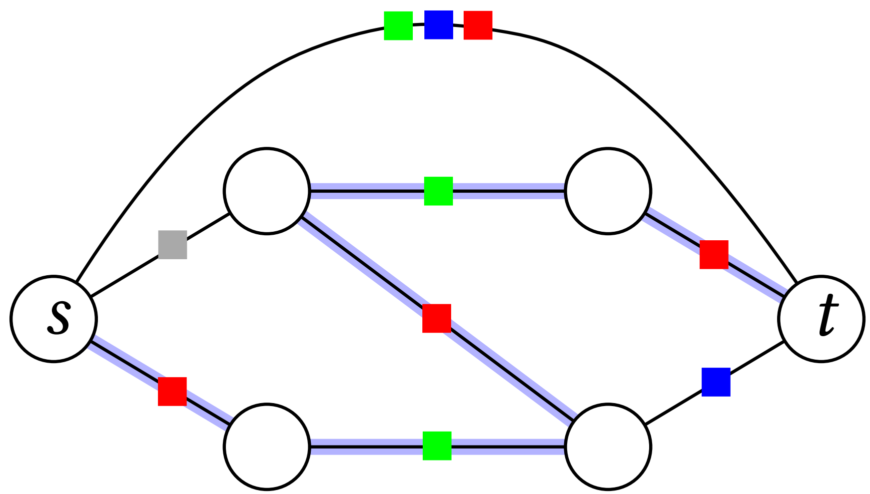

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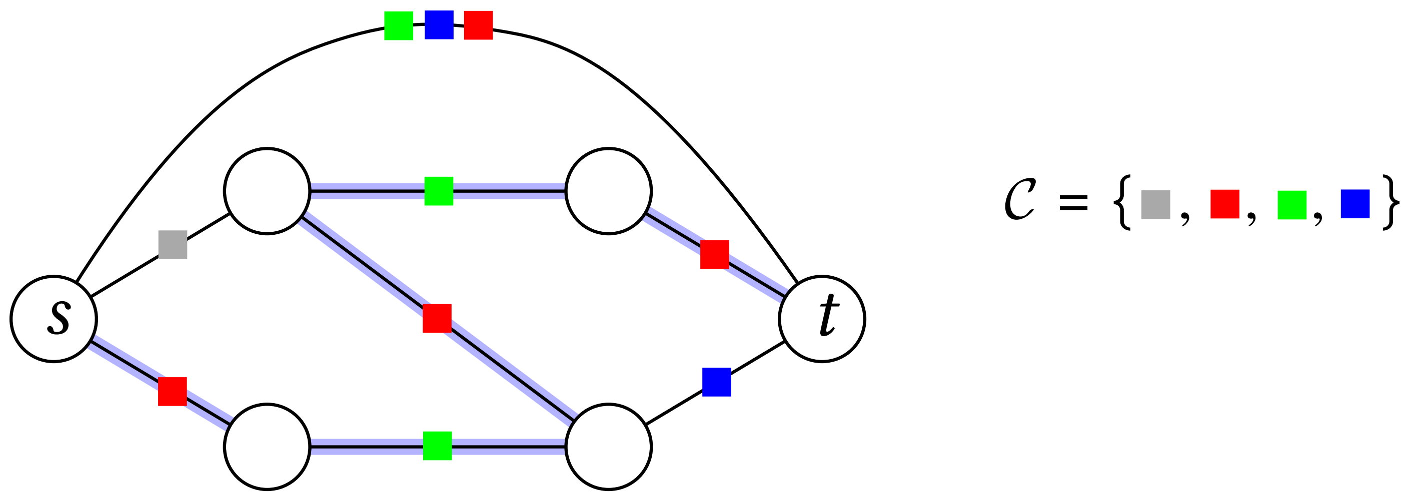

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in other words, find a path π that minimizes $\left| \bigcup_{e \in \pi} \mathcal{X}(e) \right|$

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Reliability of connections in Mesh networks (Yuan et al., INFOCOM'05)

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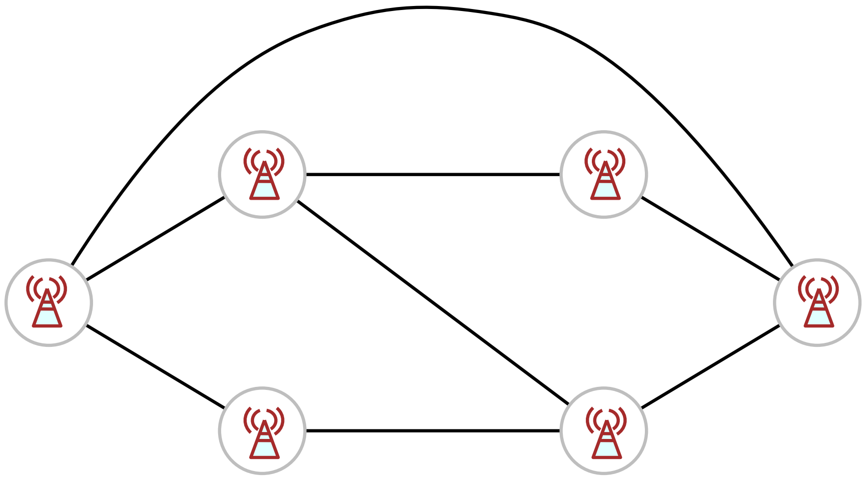

– Underlying network defines graph G

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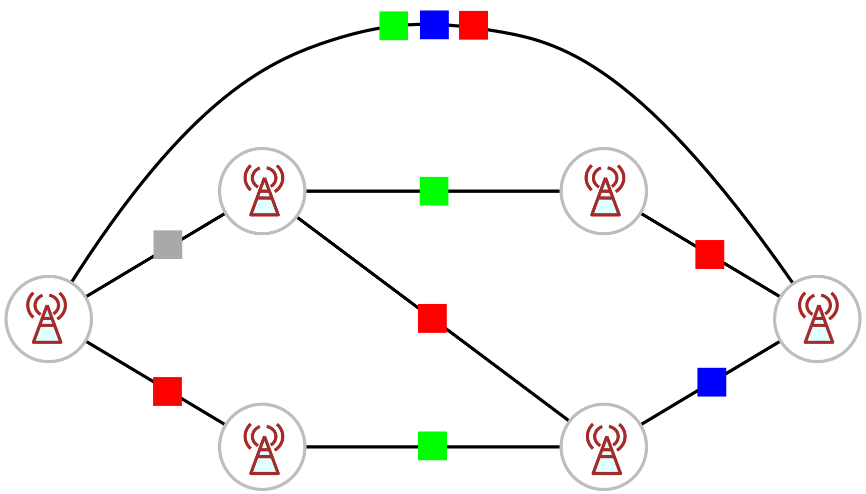

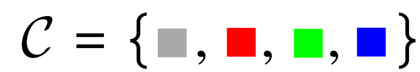

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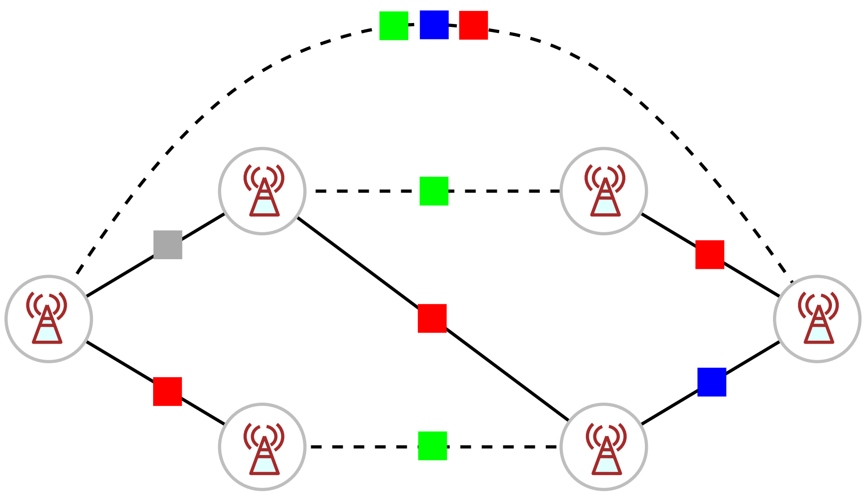

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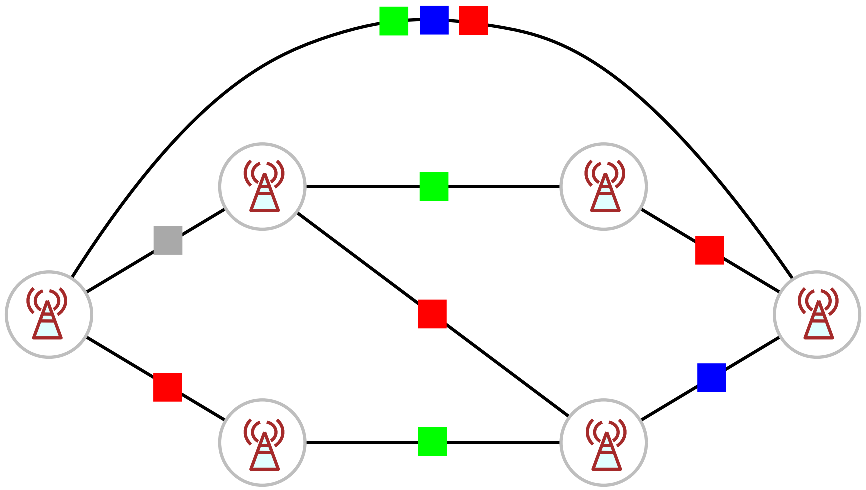

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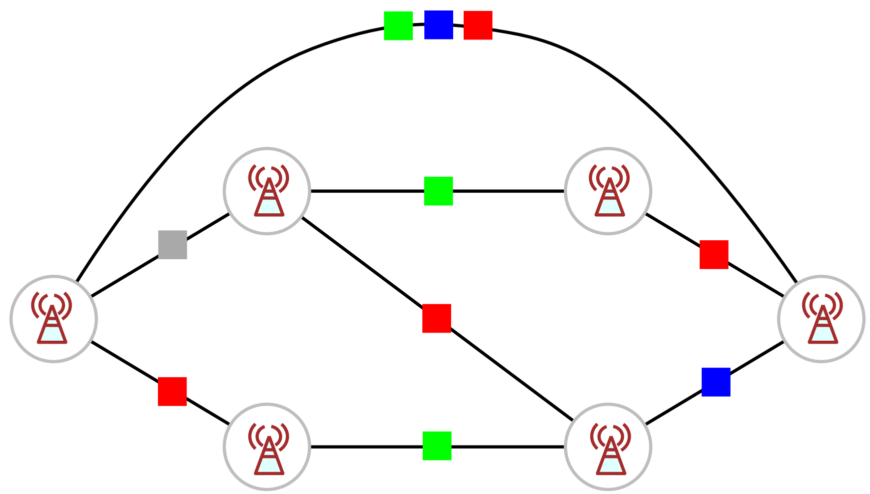

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Other Applications : Color-Cost Model

colors are services, only pay for first use!

Network Licensing costs

Truck route planning

Previous Work

- Yuan et al.' 05 : NP-hardness and heuristic algorithms
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- Improved hardness guarantees? — Conditional hardness of $o(n^{1/8})$
- Sublinear approximation? — An $O(n^{2/3})$-approximation
- Practical algorithms? — Two greedy strategies
  Significantly better performance than earlier algorithms

Summary of Results

- Hardness of approximation within a factor $O(n^{1/8})$

  Conditional hardness based on Dense Vs Random conjecture for densest k-subgraph problem

- An $O(n^{2/3})$-approximation Algorithm

  Builds upon approximation for vertex-colored graphs

- Two greedy heuristics: Greedy-Select and Greedy-Prune-Select

  Inspired from the above approximation algorithm

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Rest of this talk

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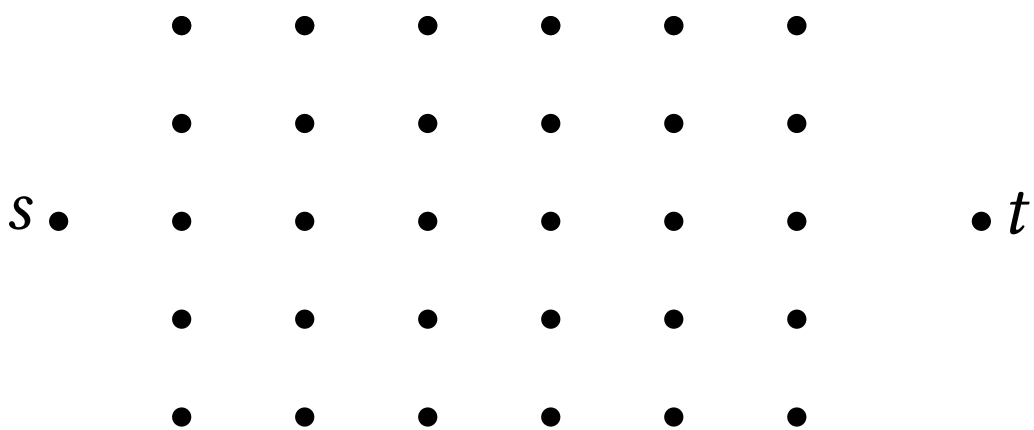

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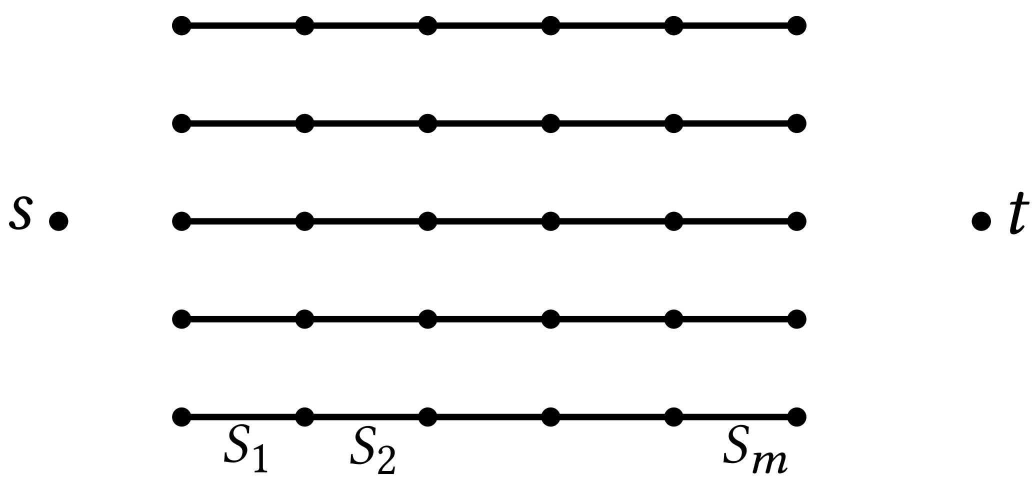

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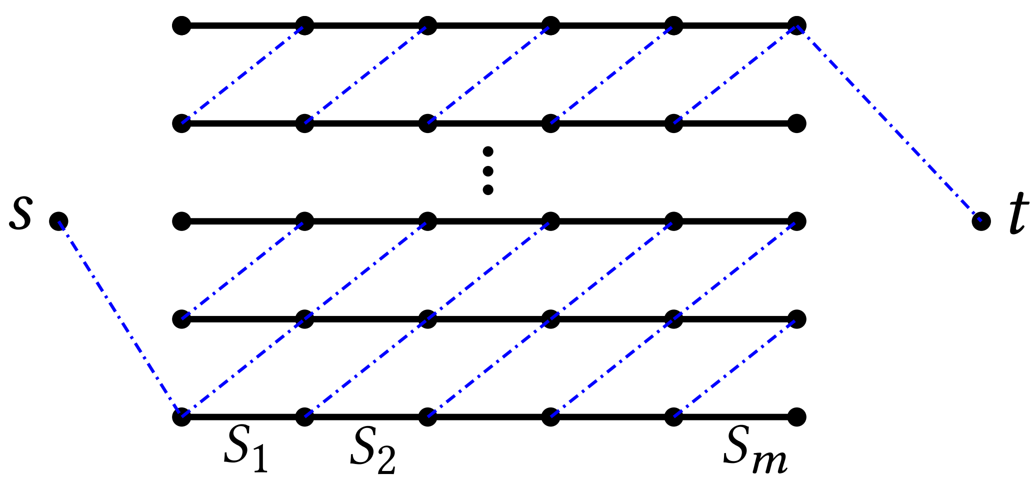

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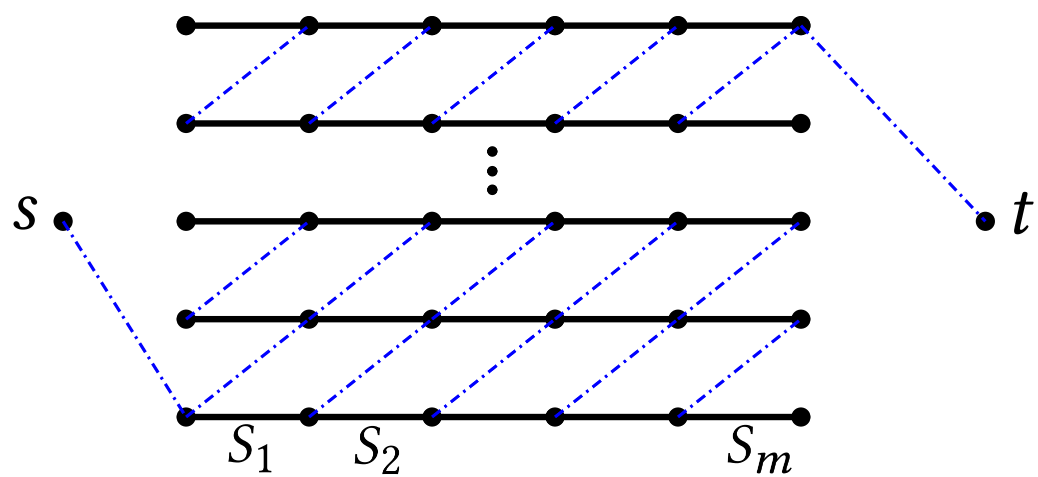

horizontal displacement m

vertical displacement $m - k$

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Minimum k-union corresponds to a min-color path in G

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diameter of a connected dense component $C_i = \frac{\text{size of } C_i}{\text{min-degree}} \leq \frac{n_i}{\beta}$

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For $\beta = n^{1/3}$, we obtain a path π that uses at most $O(n^{2/3}) \cdot k$ colors

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Greedy-Prune

–Select a color $c_{\min}$ that occurs on fewest number of edges of G

– Remove all edges containing $c_{\min}$ provided s, t are still connected

(Also edges that do not lie in same component as s, t)

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- Remove all edges containing $c_{\min}$ provided s, t are still connected

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Let G' be the graph obtained from above. Return shortest path in G'

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– Return the best path found

$c = 4$ seems to gives a good compromise between path quality and runtime

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Two major concerns:

– application scenario connectivity is typically not random

– have small diameters, small difference between dijkstra and optimal w.h.p

(less room for improvement)

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Two major concerns:

– application scenario connectivity is typically not random

– have small diameters, small difference between dijkstra and optimal w.h.p

(less room for improvement)

Instance	Dijkstra	Best known	Remarks
Layered	43.38	17.6	$k = 4$ nodes per layer
Unit-disk	34.66	13.88	$n = 1000$ random disks in a 10×100 rectangle
Road-network	366	246	$1.5M$ nodes, $2.7M$ edges, 500 colors
Uniform-col [19]	11.45	9.5	edges added with $p = \log n / 2n$

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Need to construct more challenging instances!

Constructing Challenging Instances

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For any path π of length ℓ, $p_{i\pi} = 1 - \left(1 - \frac{1}{|\mathcal{C}|}\right)^{\ell}$ (prob. that π uses color i)

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Prob of k successes in binomial trial $B(|\mathcal{C}|, p_{i\pi})$ (typically a small value if k is small w.r.t $|\mathcal{C}|$)

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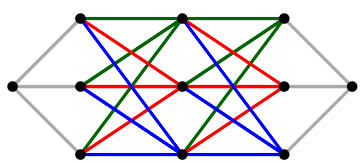
- Unit disk intersection graphs
- Layered Graphs
- CA Road network datatset
- Internet topology datatset (of ISP providers, `topology-zoo.org`)



Experimental Results

Number of colors = 50 (resp. 500) on small (resp. large) instances,

Table 2. Path quality and running time on layered graph instances.

Algorithm	Colors used	Time taken (ms)	Colors used	Time taken (ms)
	$4 \times 125 = 0.5k$ nodes		$4 \times 2500 = 10k$ nodes	
Dijkstra	36.8	0.6	441.8	23.6
SPACOA	33.6	65	396	127×10^3
Greedy-Select	18.2	12.6	185.6	3.5×10^3
Greedy-Prune-Select	17.2	49	173	12.5×10^3
ILP	16.4	707×10^3	∞	∞

Table : Layered graph instances

SPACOA is best previous heuristic

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Dijkstra	28.8	1	357.8	38
SPACOA	23	124.8	333.6	41.4×10^3
Greedy-Select	14.2	13	145.6	4.7×10^3
Greedy-Prune-Select	13.4	55	134	17.6×10^3
ILP	12.6	1176×10^3	∞	∞

Table : Unit-disk Graph instances

SPACOA is best previous heuristic

Results Contd.

Table 4. Path quality and running time on some real world instances.

Algorithm	Colors used	Time taken (ms)	Colors used	Time taken (ms)
	CA Road Network		Internet topology	
Dijkstra	366	3.068×10^3	7	26
SPACOA	355	3.12×10^6	4	3111
Greedy-Select	251	0.73×10^6	5	29
Greedy-Prune-Select	246	2.71×10^6	4	286
ILP	∞	∞	4	1817

CA Road network 1.5 M nodes, 2.7M edges, 500 colors randomly assigned to edges

Internet topology 5.6k nodes, 8.6k edges, 261 colors (each color is a service provider)

Table 5. Path quality and running time on Uniform-Col instances.

Algorithm	Colors used	Time taken (in ms)	Colors used	Time taken (in ms)
	10^3 nodes		10^4 nodes	
Dijkstra	11.45	0.95	11.7	15
SPACOA	9.95	75.45	11.2	8664
Greedy-Select	10.4	9.2	11.5	150
Greedy-Prune-Select	10.3	46.3	11.4	1919
ILP	9.5	3913.8	-	-

Table: Random Colored Graphs

Summary of Results

- Hardness of approximation within a factor $O(n^{1/8})$
- An $O(n^{2/3})$-approximation Algorithm
- Two greedy heuristics: Greedy-Select and Greedy-Prune-Select
- Experimental evaluation and benchmark datasets

Thanks!

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Thanks!

Rest of this talk