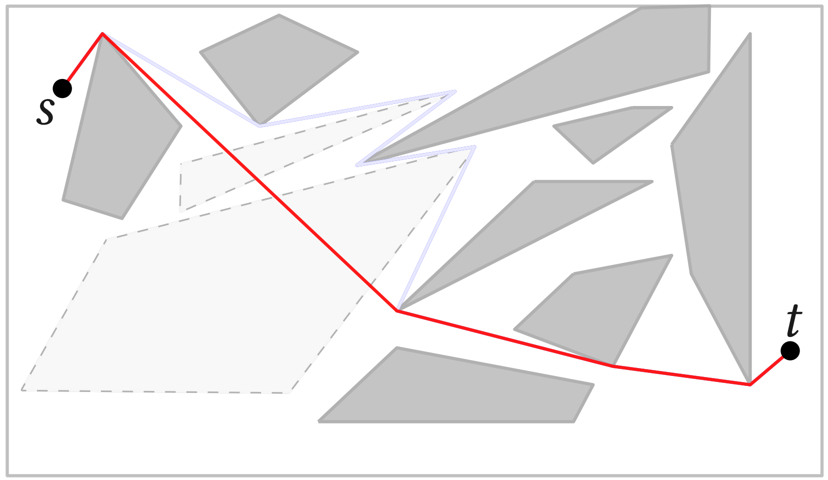
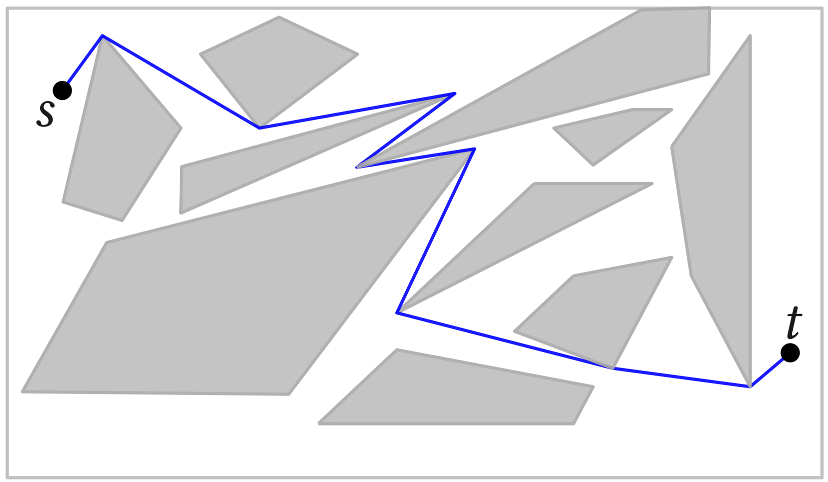


Computing Shortest Paths in the Plane with Removable Obstacles

Pankaj K. Agarwal, Neeraj Kumar, Stavros Sintos and Subhash Suri

Duke University and UC Santa Barbara

Problem Description



Input: h polygonal obstacles with n vertices, source s and target t

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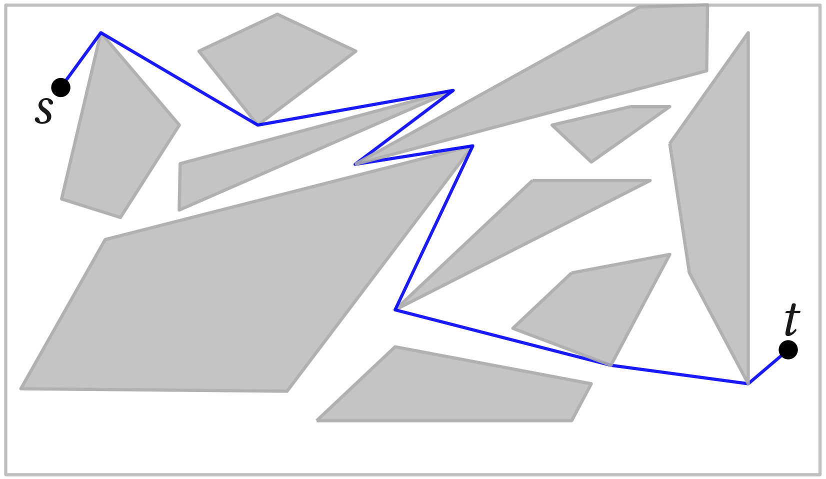

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An obstacle P_i can be removed by paying cost c_i

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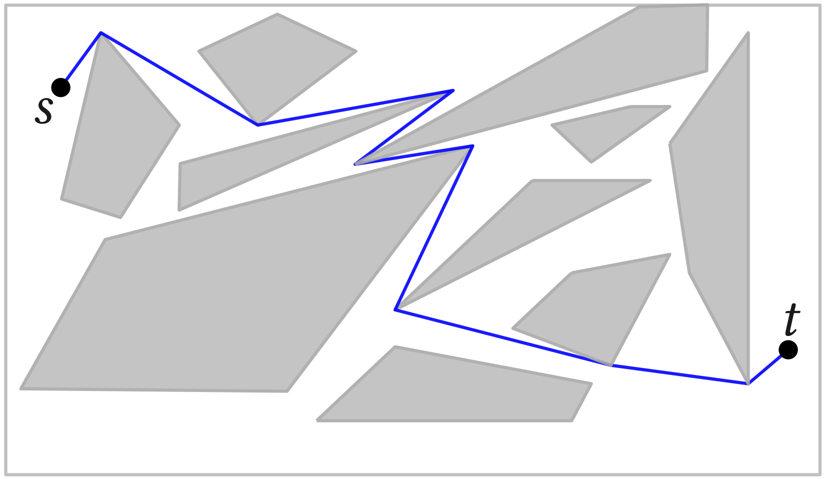

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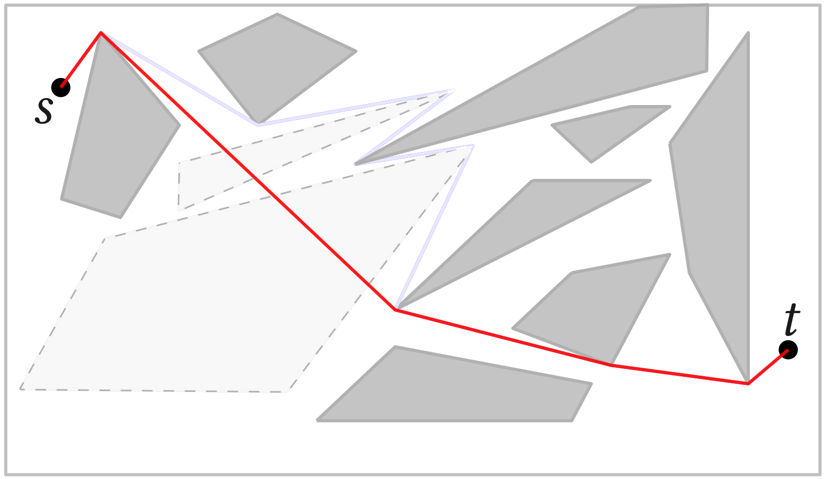

Input: h polygonal obstacles with n vertices, source s and target t

An obstacle P_i can be removed by paying cost c_i

Given a cost budget C, which obstacles should we remove

so that length of path from s to t is minimized?

Assumption: Obstacles are mutually disjoint and convex

Motivation

Classically, we deal with finding shortest path in a fixed network

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What if we are allowed to modify the environment

What changes should we make?

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Applications

Urban Planning

Adding ‘shortcuts” to ease congestion

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Classically, we deal with finding shortest path in a fixed network

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Applications

Reconfiguring road networks

– such as by adding flyovers

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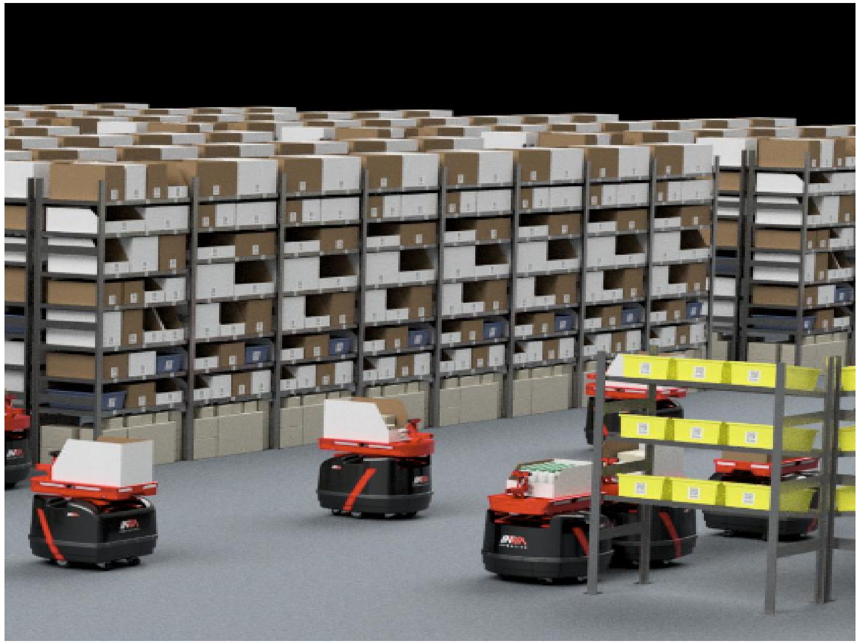
Applications

Re-organize Warehouse Layout

– Shorten frequent paths for robot



Path Planning Under Uncertainity

Obstacle P_i is present in input with independent probability β_i,

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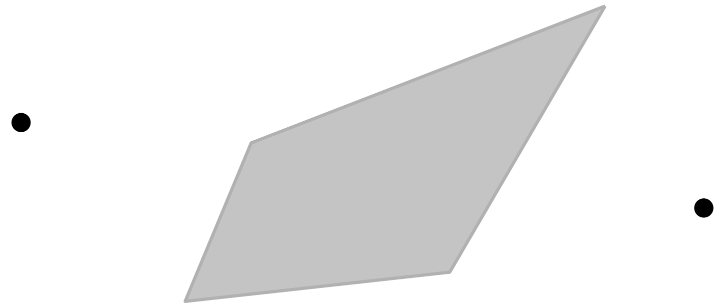

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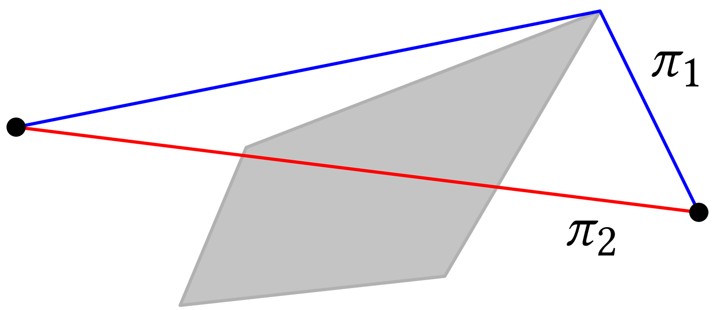

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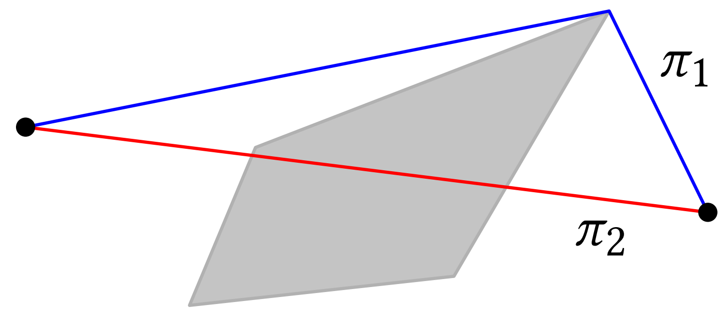

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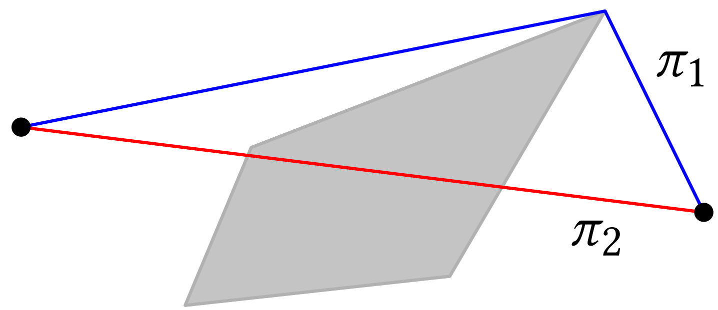

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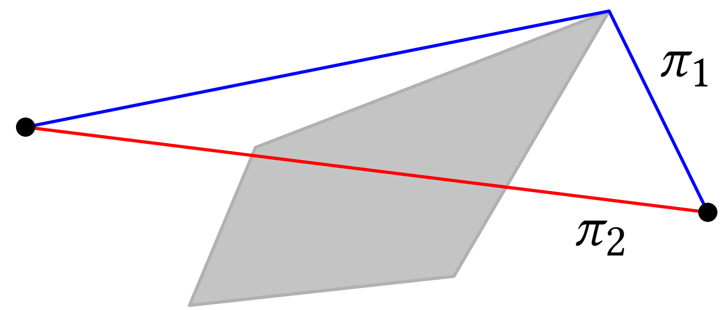

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Can reduce to the earlier cost-based model by taking negative logarithms

Related Work

[HKS'17] Shortest Paths in the Plane with Obstacle Violations

Computes shortest s–t path in the plane that removes at most k obstacles
$O(k^2 n \log n)$ algorithm using Continuous Dijkstra

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Obstacles have unit cost of removal, budget = k

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Cardinality Model
Obstacles have unit cost of removal, budget = k

We study the more general **cost-based model** of obstacle removal

Our Results

- Problem of computing a path of length at most L and cost at most C is NP-HARD even for vertical segment obstacles.

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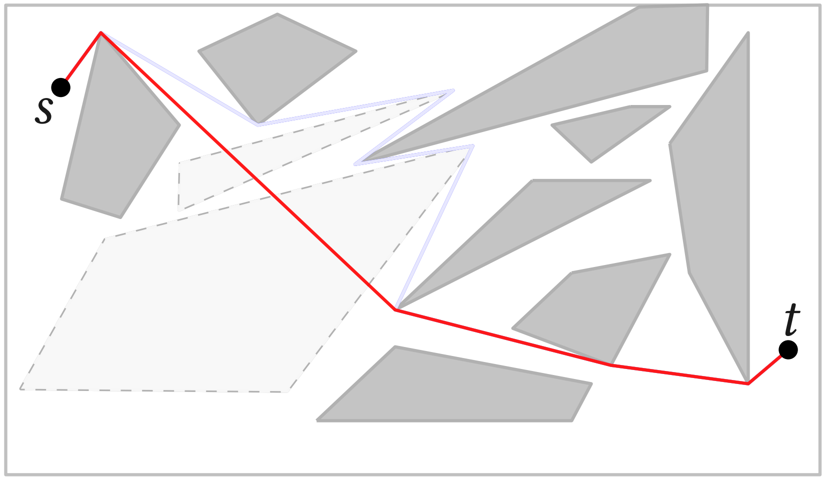
Rest of the talk

A Simple FPTAS



Observations

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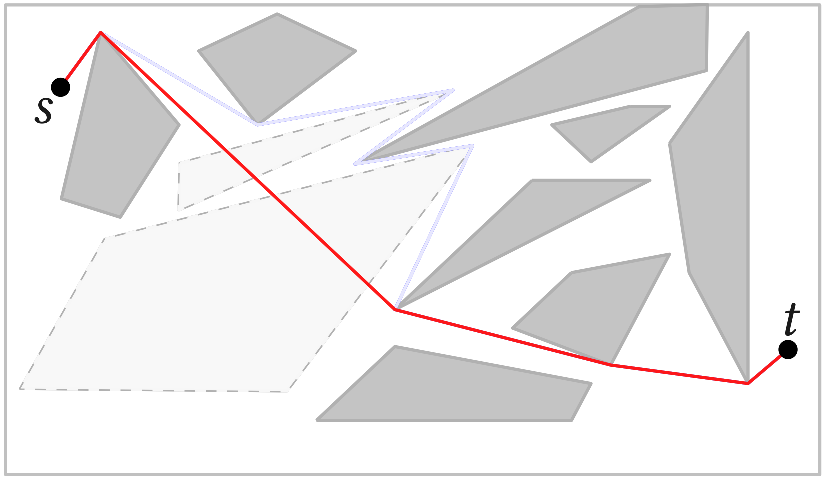

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- Since π has minimum length, it must only turn at obstacle vertices

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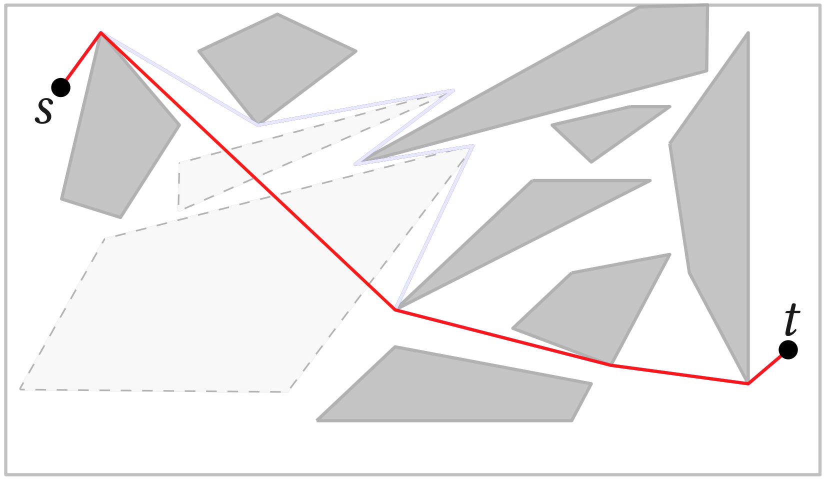

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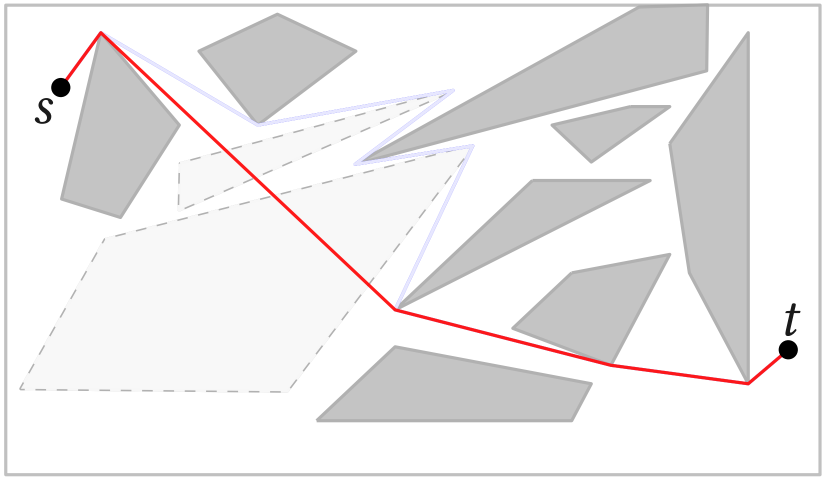

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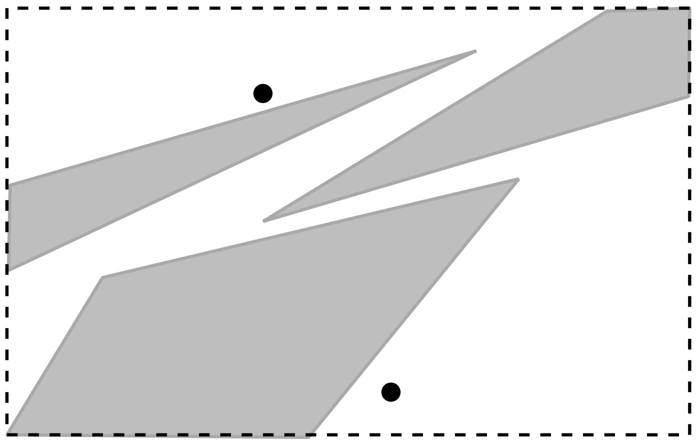
Scale all costs such that budget $C = h$, the number of obstacles



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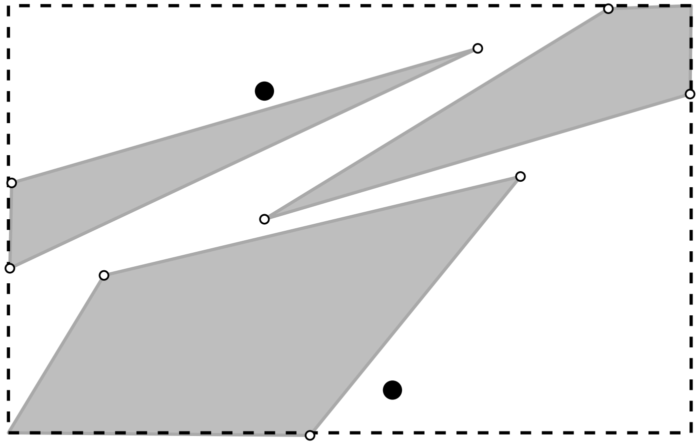
Model as a graph problem:



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A Simple FPTAS

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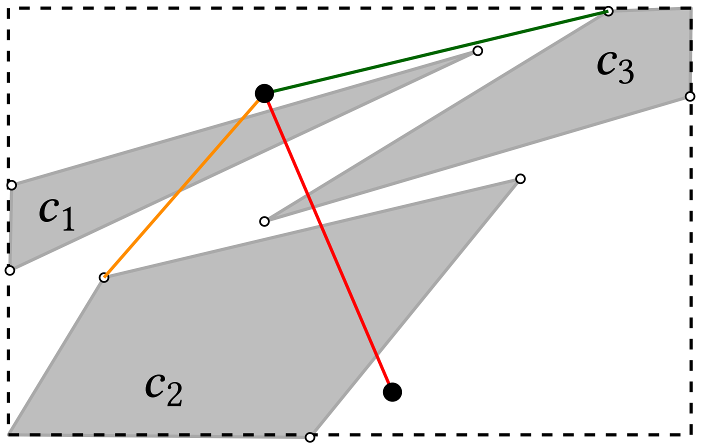

Cost of red edge is $c_1 + c_2 + c_3$

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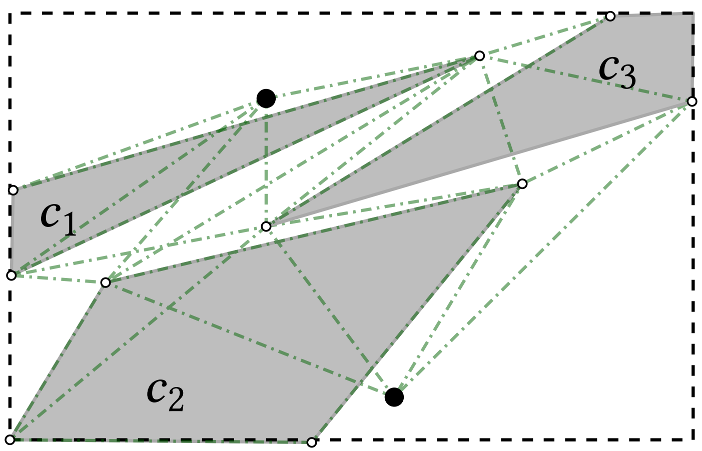

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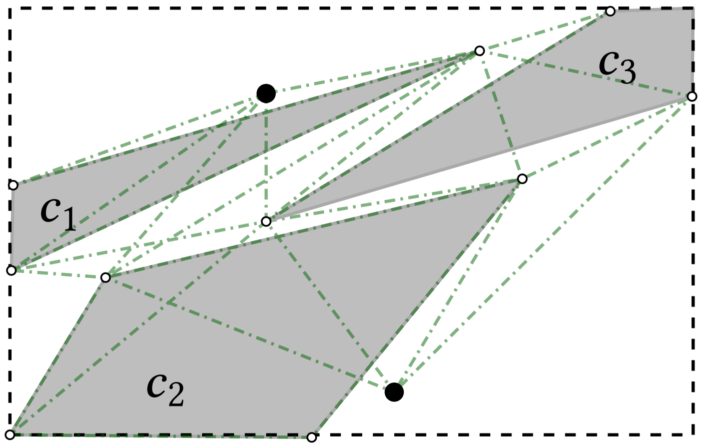

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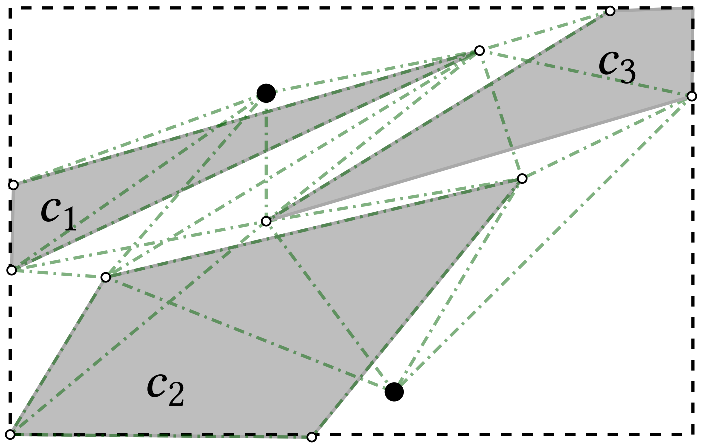

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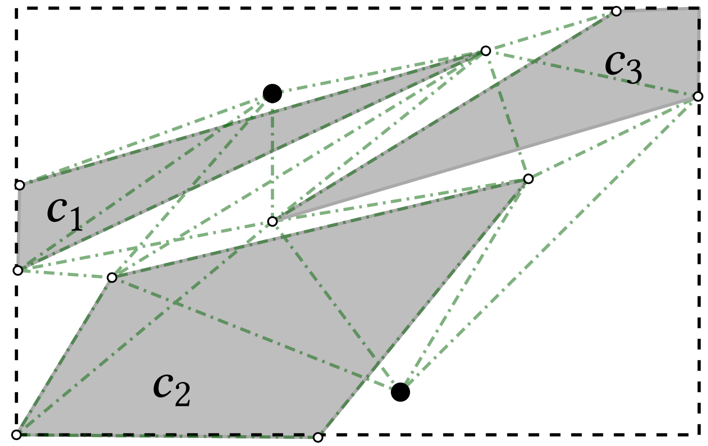

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$(1+\epsilon)$-approximation using Dijkstra's algorithm in $\tilde{O}(\frac{n^2 h}{\epsilon})$ time by creating $O(\frac{h}{\epsilon})$ copies of vertices and edges

A Faster FPTAS

Key idea is to construct a sparse **Viability Graph**

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Preserves pairwise distances within a factor of $(1+\epsilon)$

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Primary challenge is to construct the graph VG_1

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Inspiration: L_1 Visibility Graphs : [Clarkson-Kapoor-Vaidya'87]

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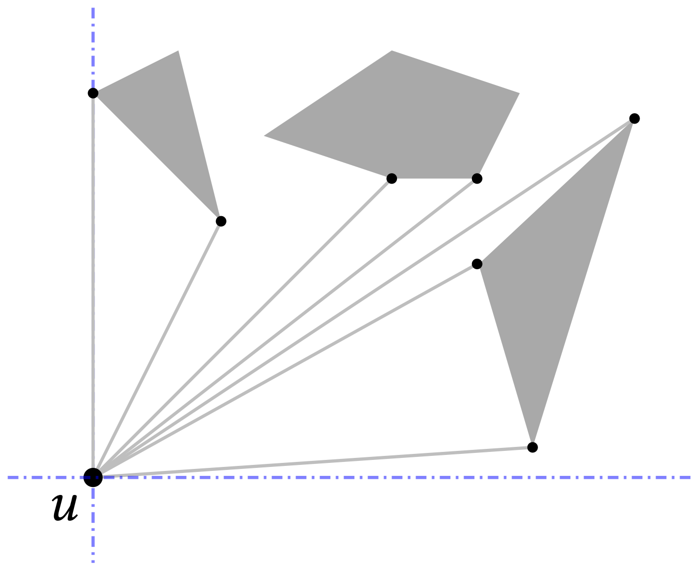

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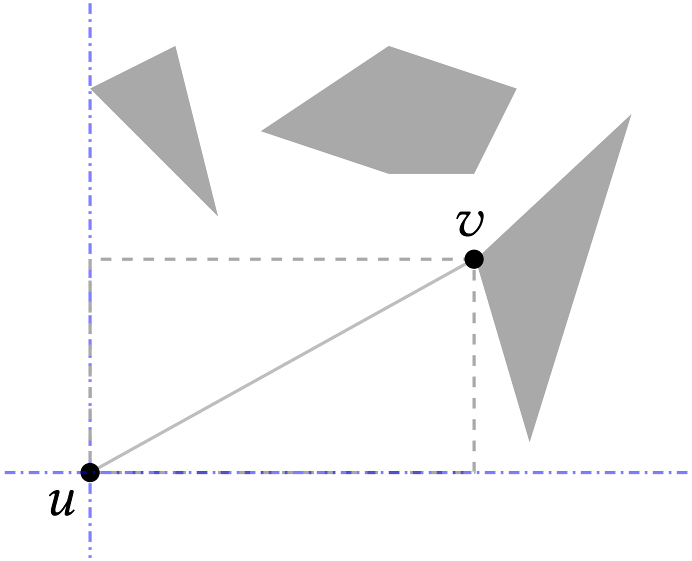

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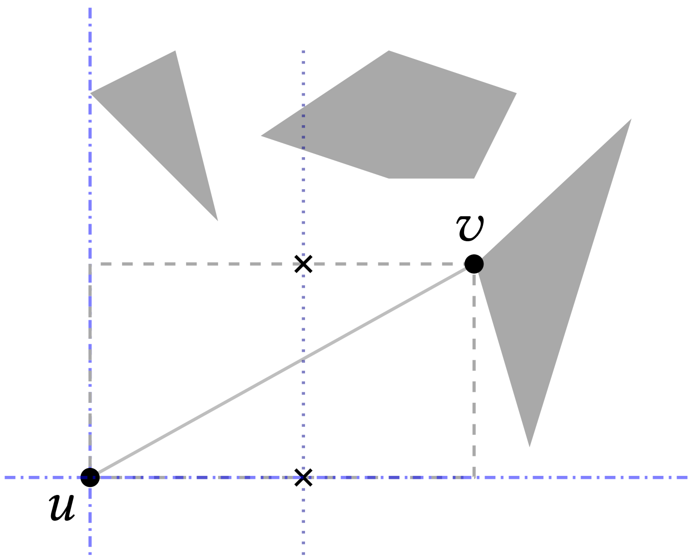

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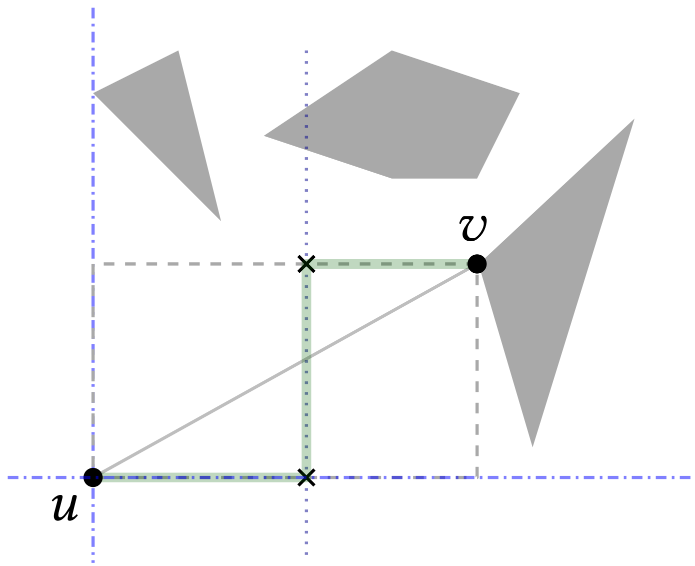

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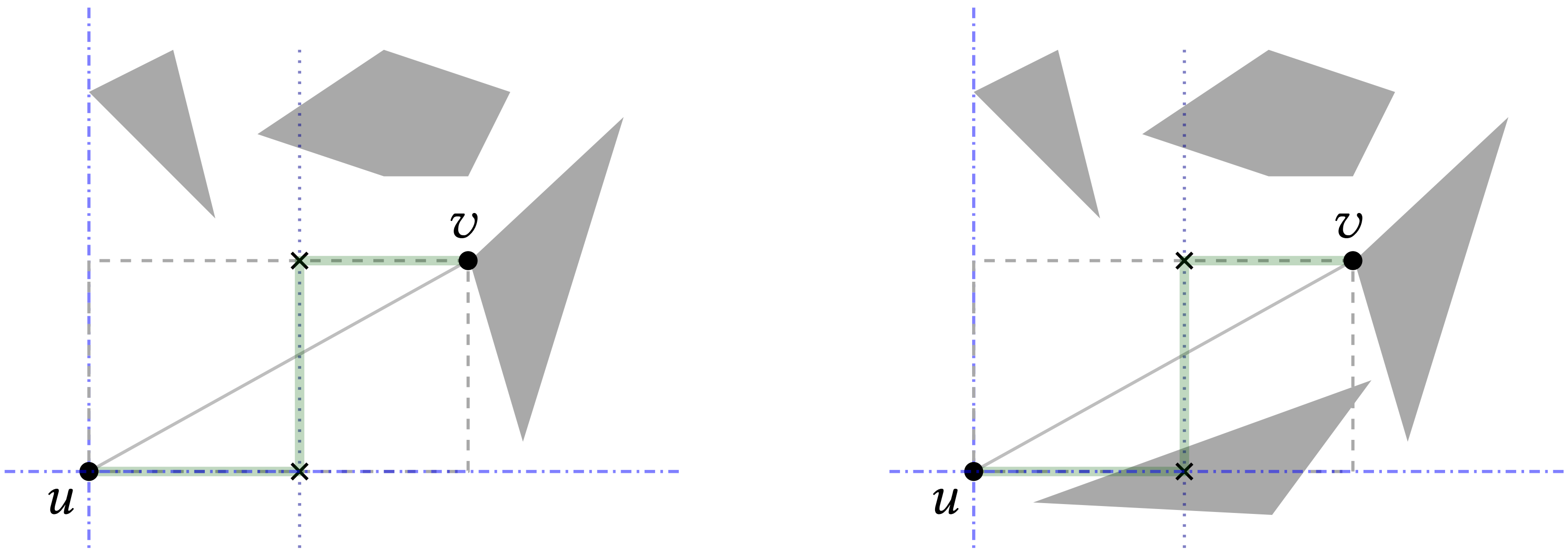

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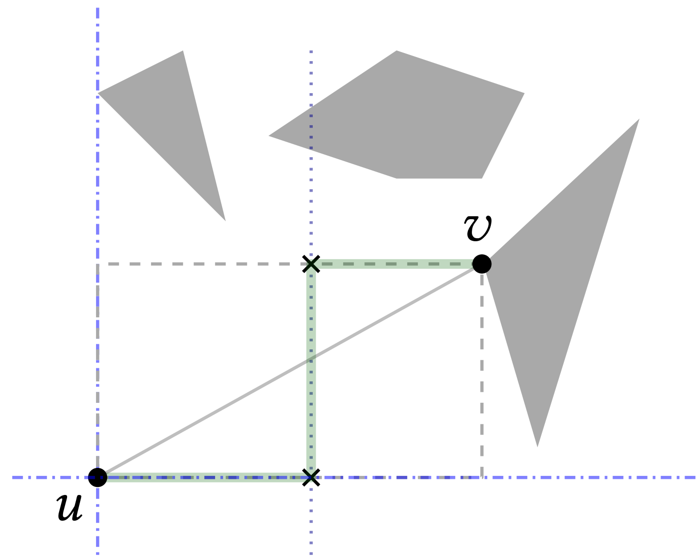

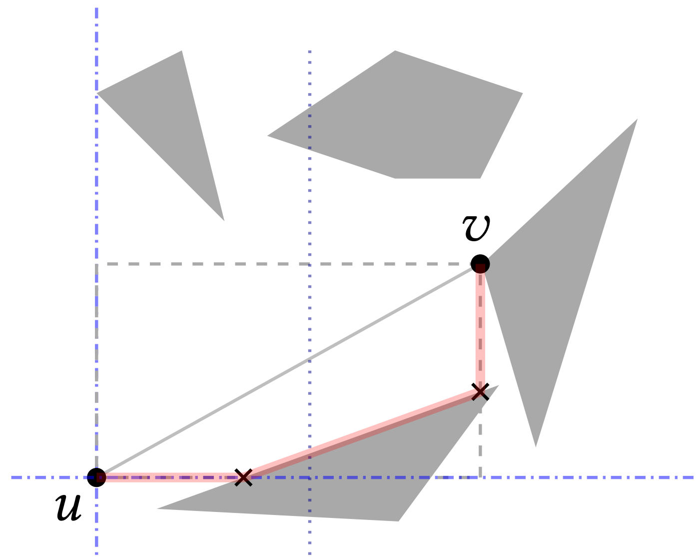

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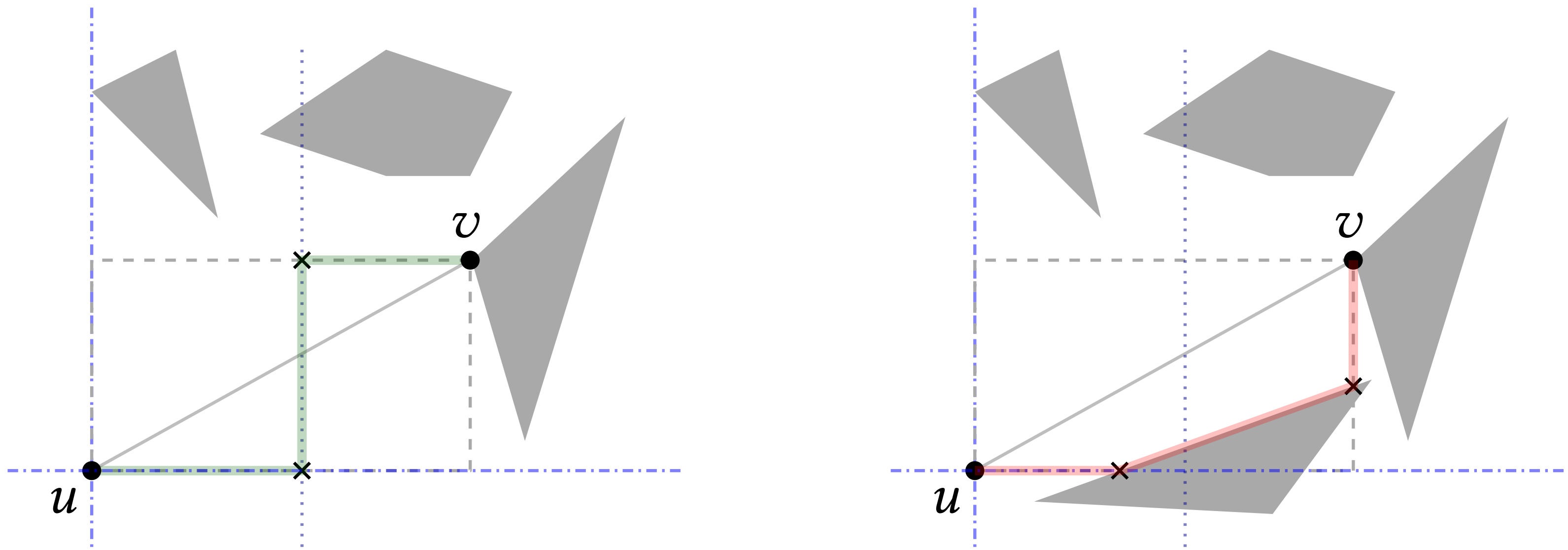

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We show how to construct $O(n \log n)$ **size** L_1 **Viability Graphs**

edges can go through obstacles

Algorithm Overview

- Will reuse idea of taking split line projections for [CKV'87]

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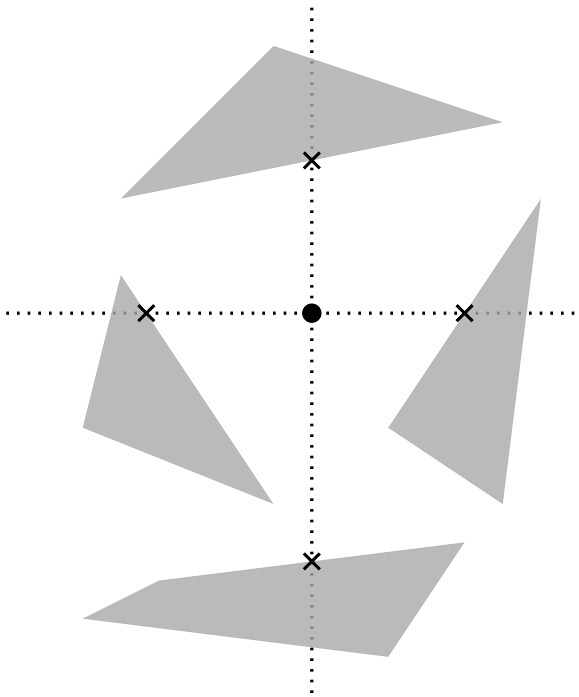

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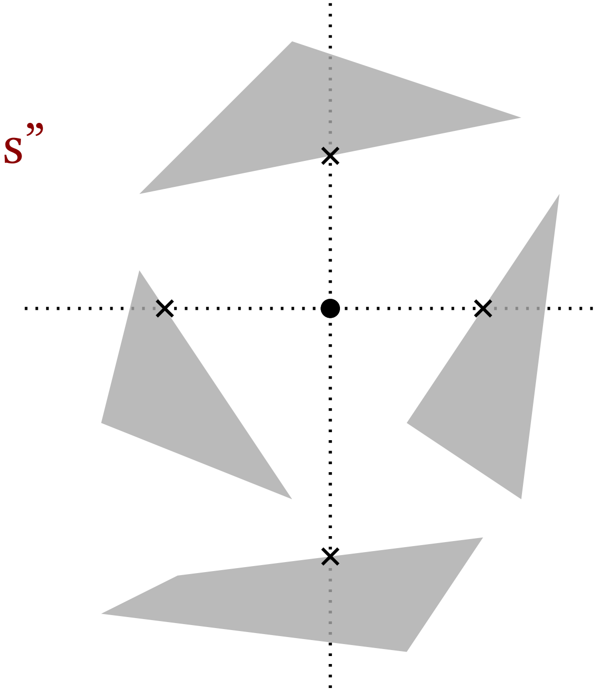

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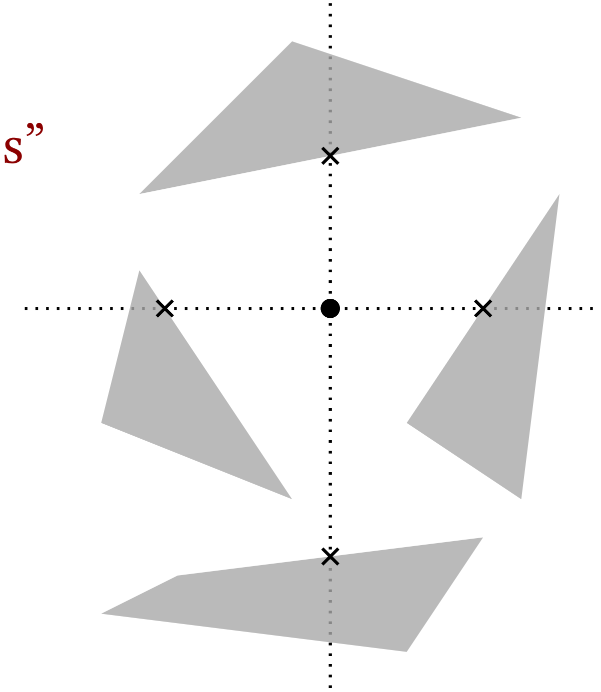

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Next we describe an algorithm for constructing an L_1 **viability graph** VG_1

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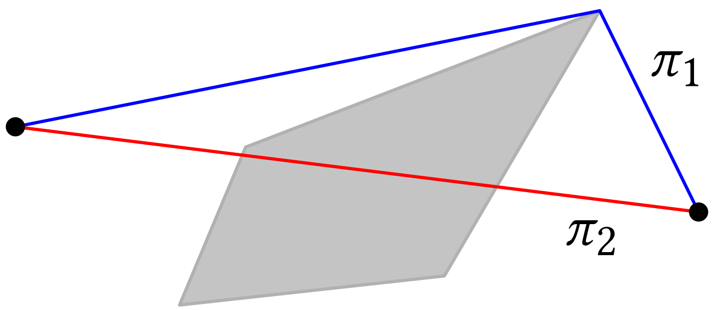
Obstacles are **convex** and **disjoint**

⇒ Shortest paths intersect zero or exactly two sides of an obstacle



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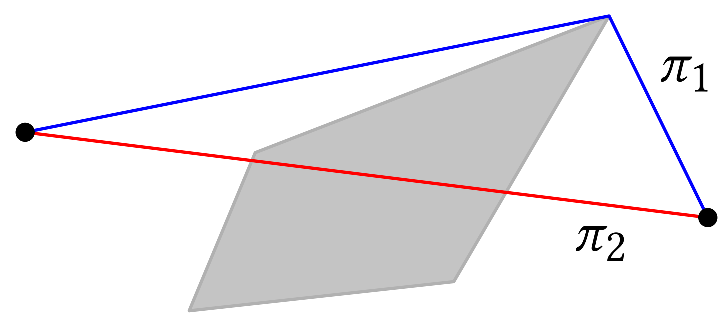

Assign cost $c_i/2$ to all segments of an obstacle that has cost c_i

Obstacles with Costs ⇒ Segments with Costs

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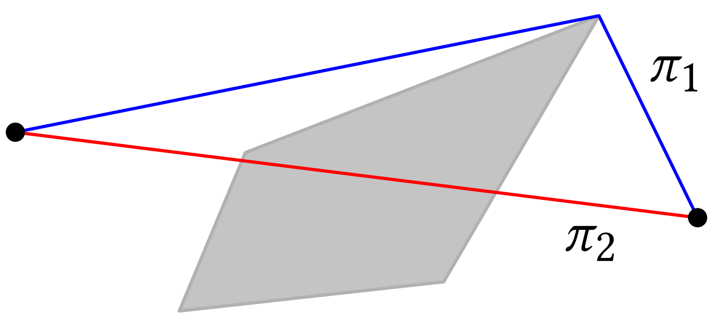
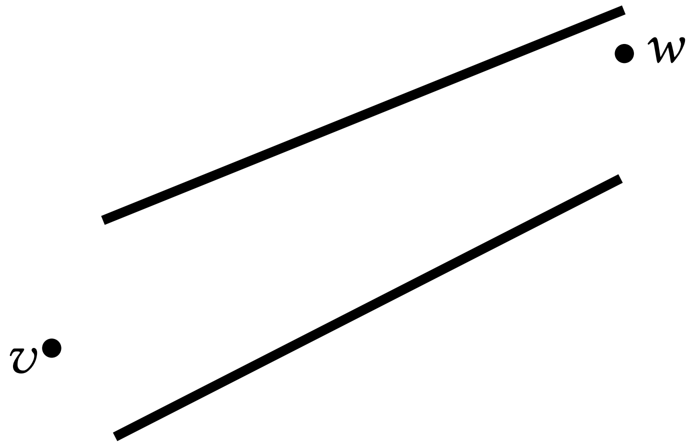


Assign cost $c_i/2$ to all segments of an obstacle that has cost c_i

Allows us to reason about geometry of line segments

Algorithm Description



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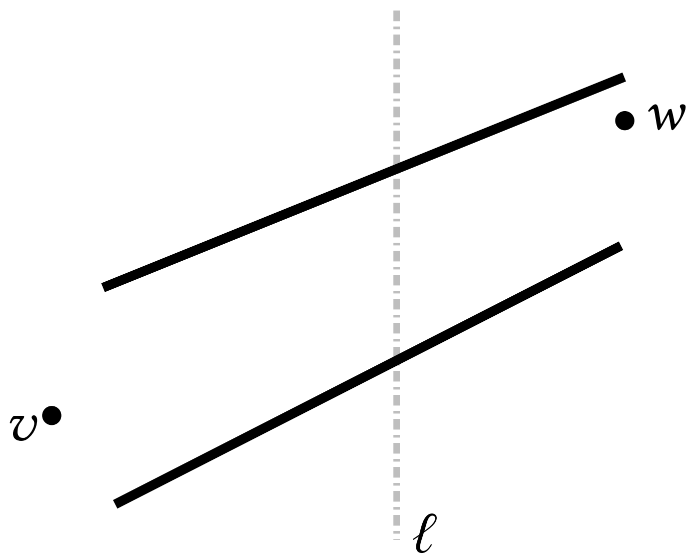
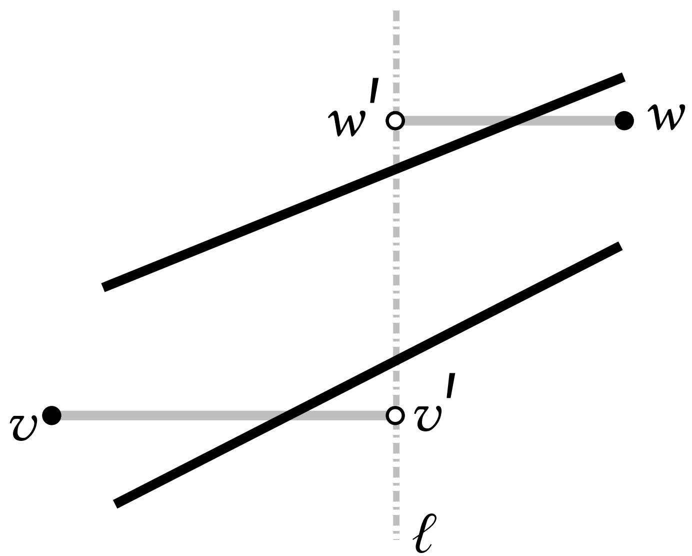


- Find the split line ℓ that splits vertices into equal sized sets V_l and V_r

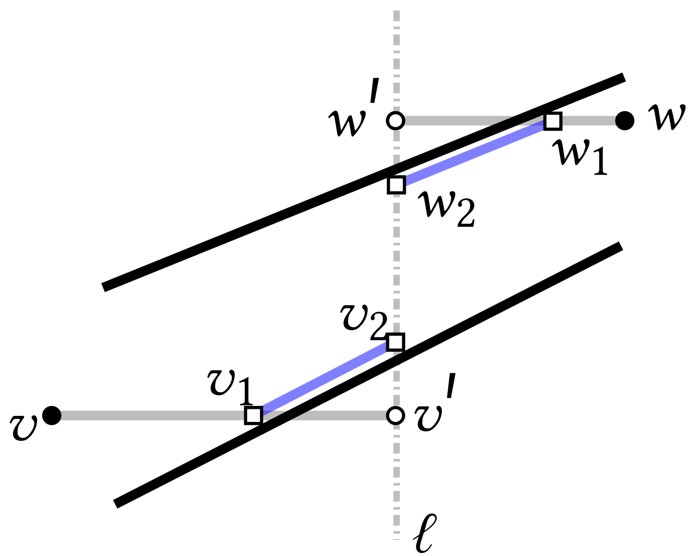
Algorithm Description



– Cost of **projection segment** is the sum of costs of all obstacle segments that it intersects

- Find the split line ℓ that splits vertices into equal sized sets V_l and V_r
- Let v' be the projection of v on this split line ℓ
  - Add vertex v' and edge vv' with cost $c(vv')$

Algorithm Description



– Cost of **projection segment** is the sum of costs of all obstacle segments that it intersects

– Bypass edges $v_1 v_2$ have cost $c(v_1 v_2) = 0$

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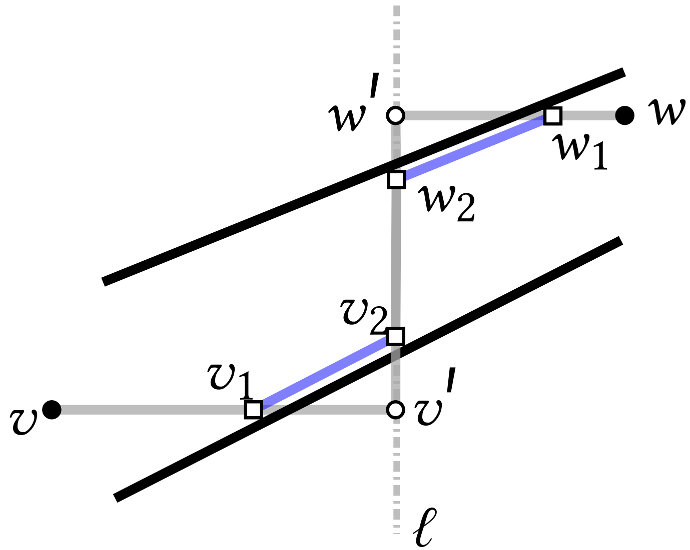

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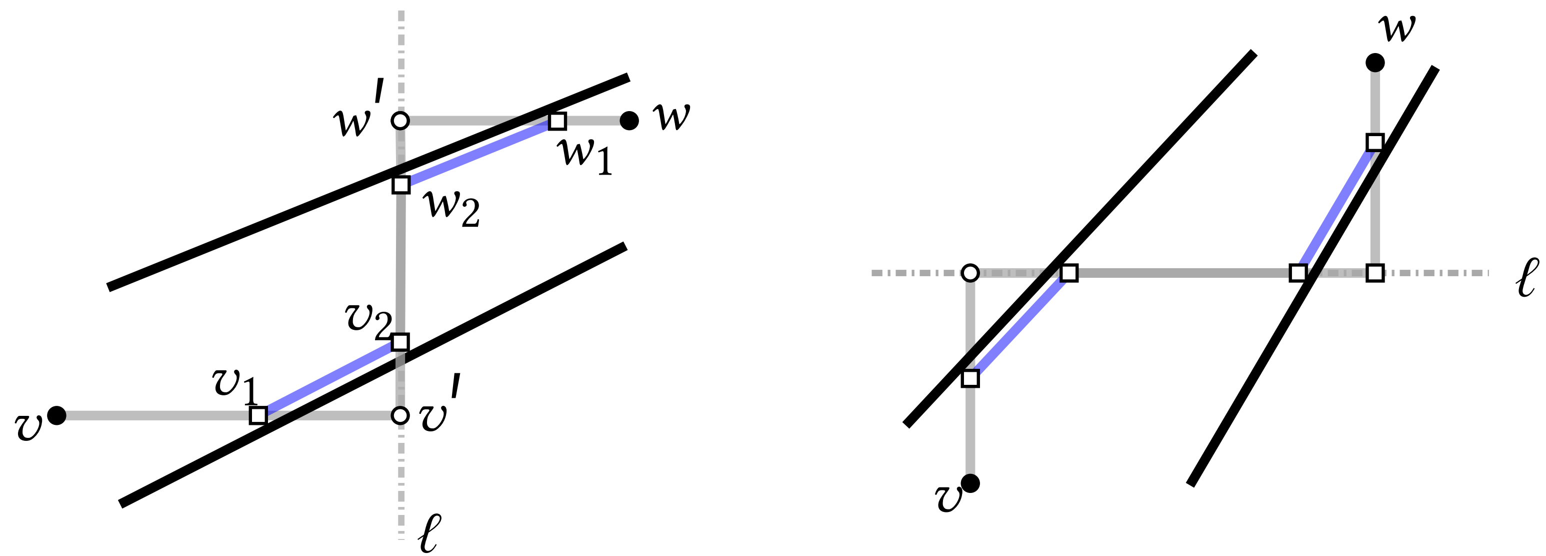

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- Connect consecutive vertices on ℓ and obstacle boundary
- Repeat for a horizontal split line. RECURSE on sets V_l and V_r.

Correctness

Suffices to show: for any pair of obstacle vertices p, q there exists a path π_{pq} such that the L_1 length $\|\pi_{pq}\|_1 \leq \|pq\|_1$ and cost $c(\pi_{pq}) \leq c(pq)$

Proof Idea:

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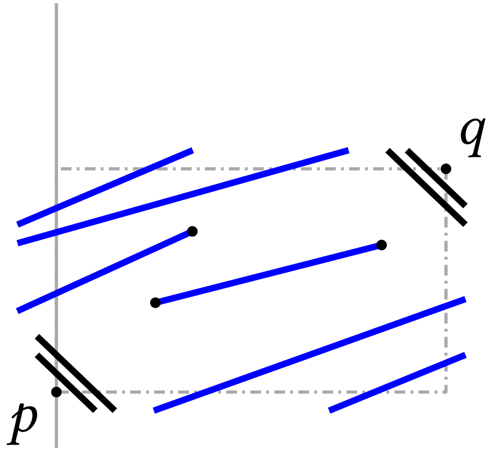

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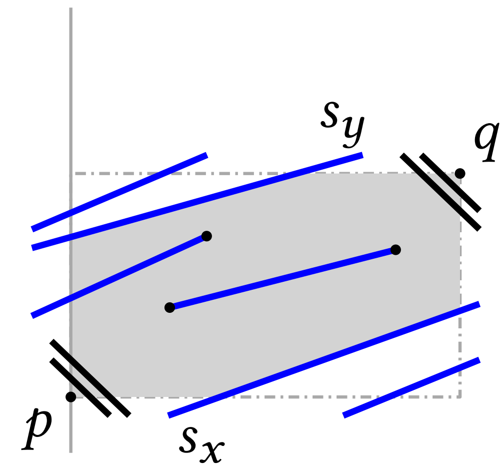

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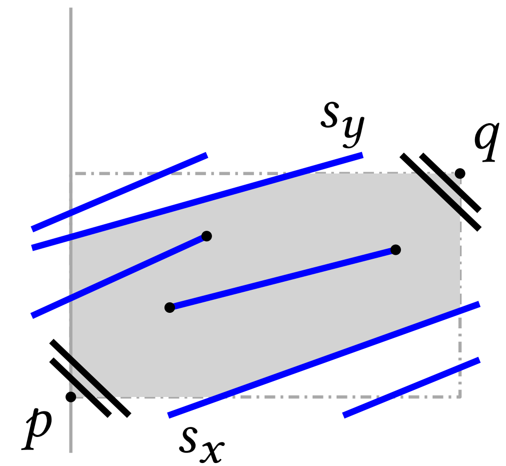
– If R_{pq} contains an obstacle vertex

– If R_{pq} does not contains an obstacle vertex



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(Induction Step)

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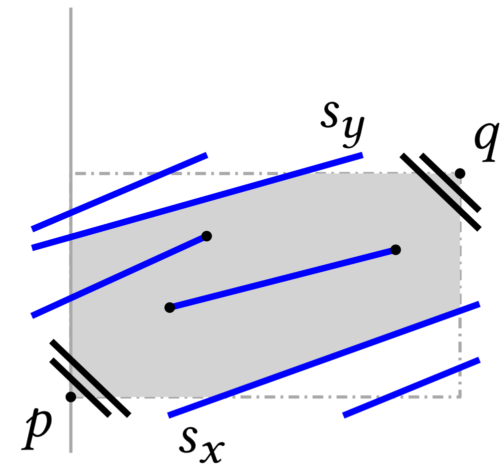
(Base Case)



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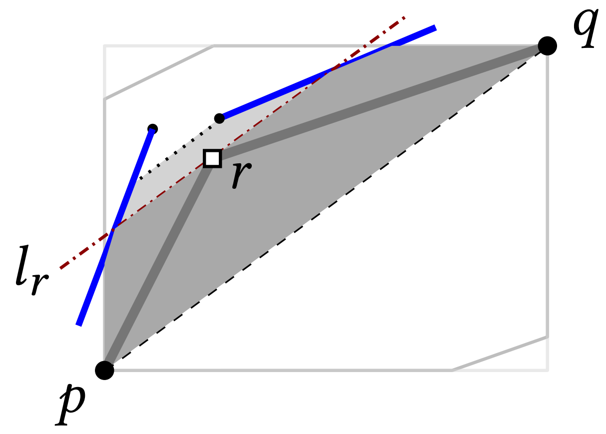

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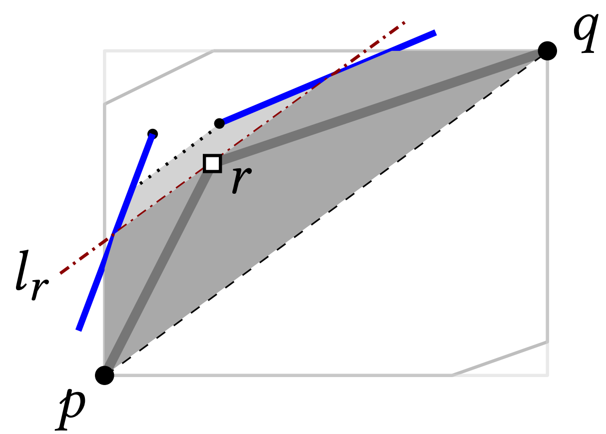

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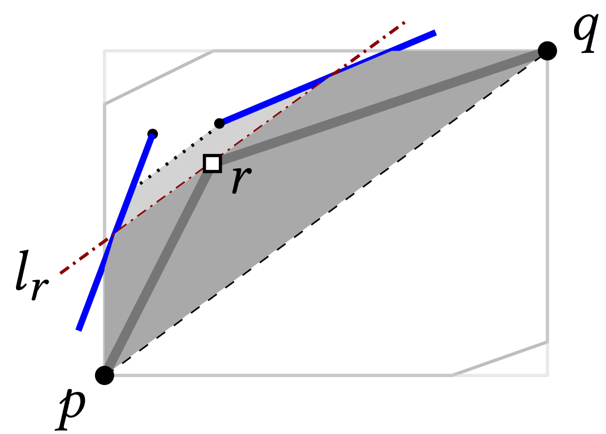

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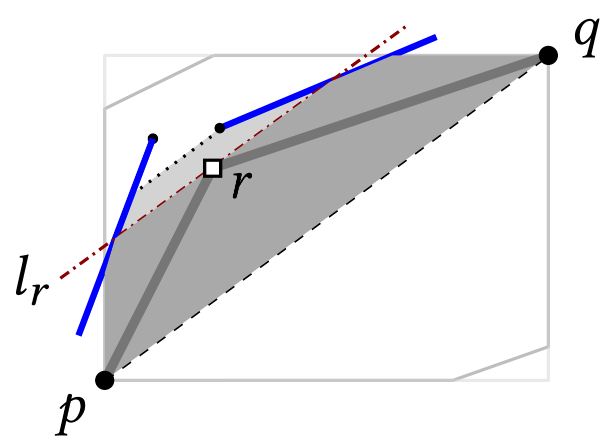

We show how to find an **intermediate vertex** r such that

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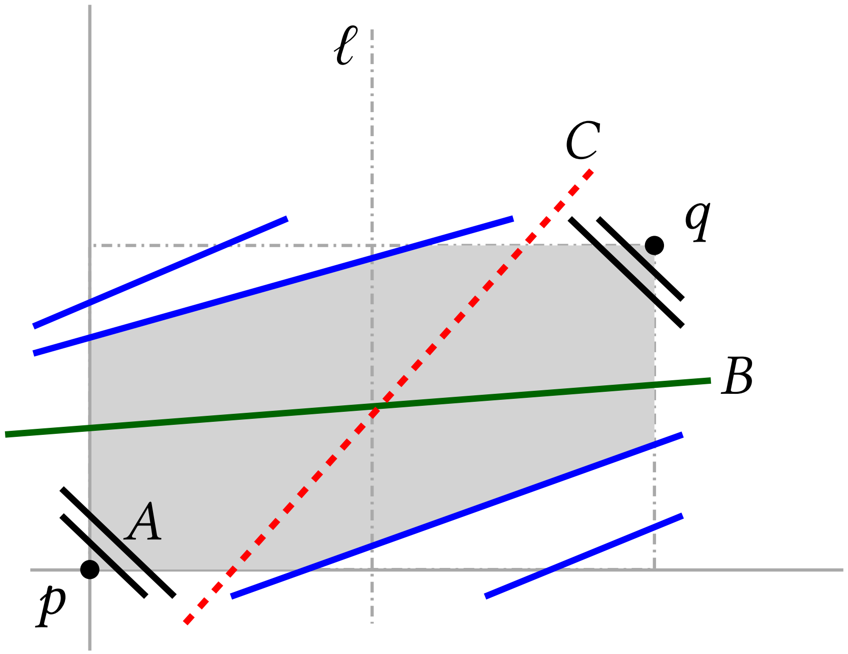
Details are technical, relies heavily on **disjointness of obstacle segments**

Base Case



Identify three types of edges A, B and C

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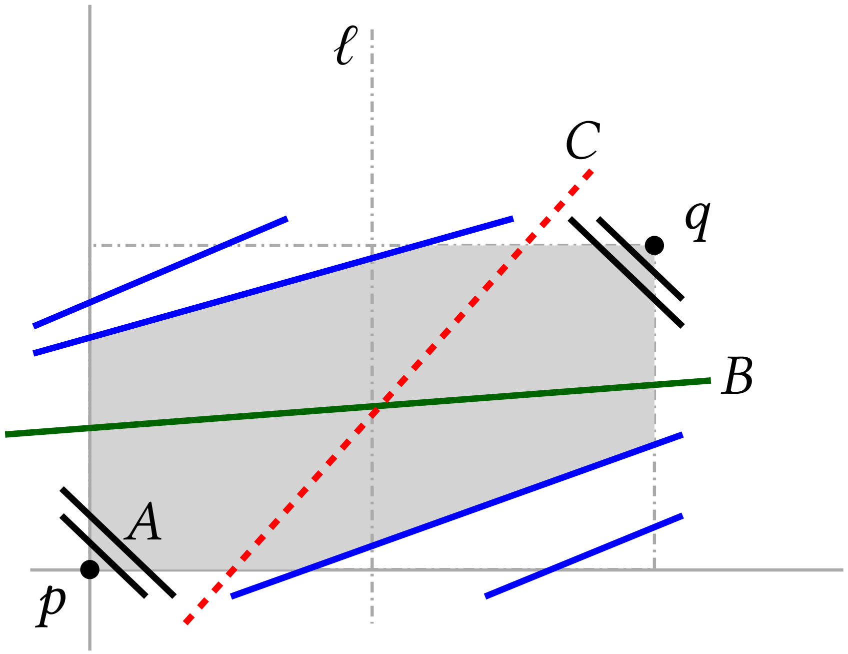

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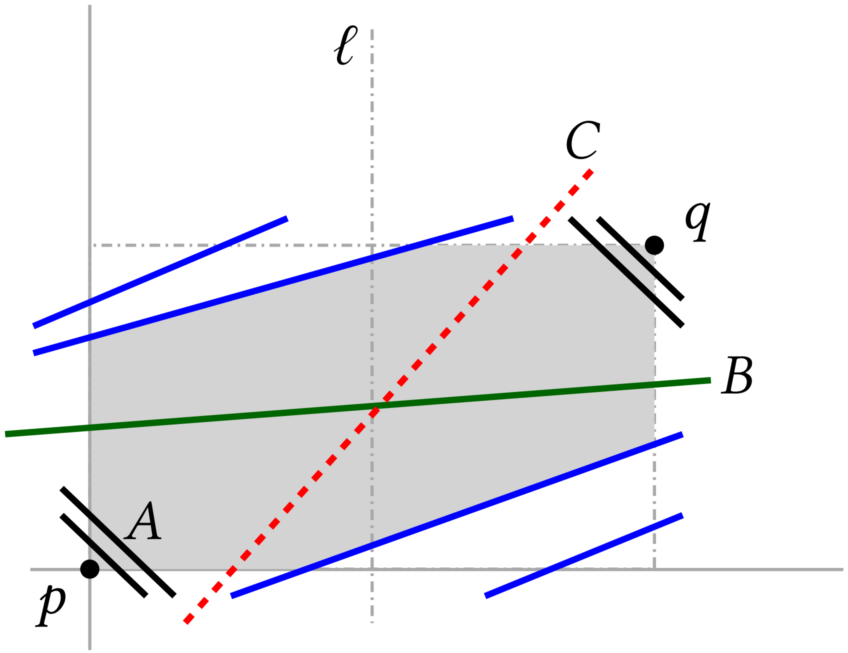

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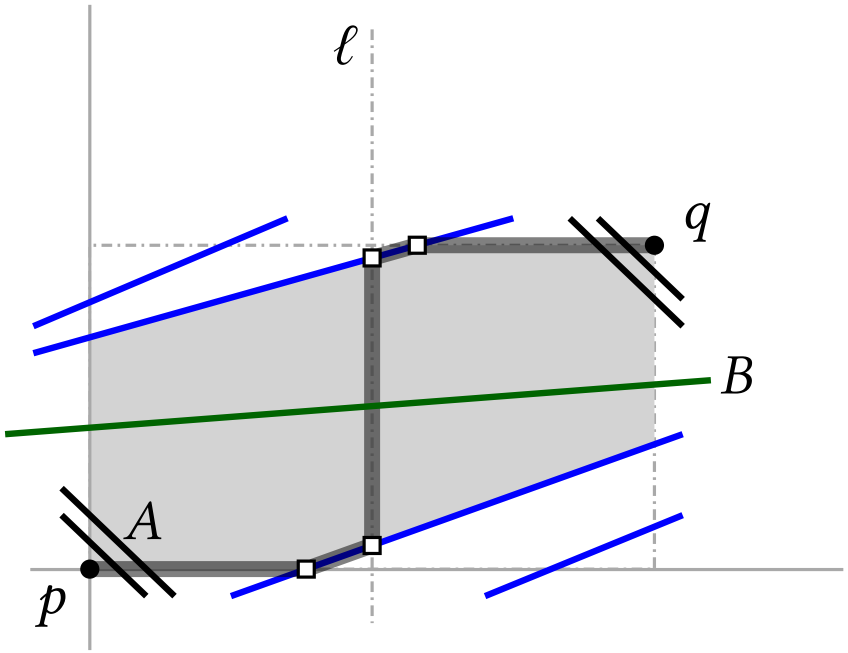

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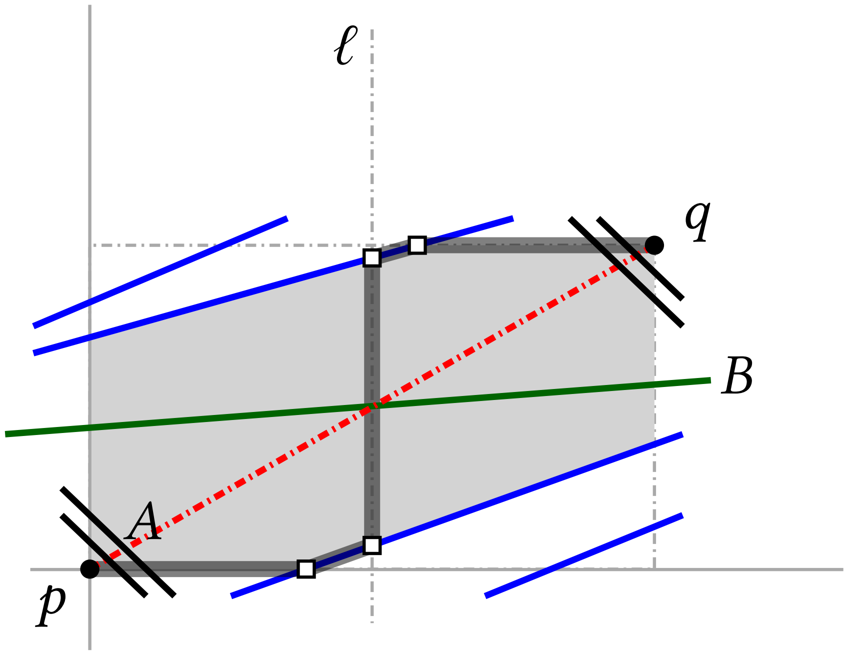

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Exists a path π_{pq} in VG_1 using projection and bypass vertices

Easy to verify that π_{pq} has same L_1 length and cost as the segment pq

A Faster FPTAS

Key idea is to construct a sparse **Viability Graph**

– trade a factor $(1+\epsilon)$ in path length for efficiency

Roadmap

– Construct an $O(n \log n)$ size viability graph VG_1 preserving L_1 distances

– Create $1/\epsilon$ copies of VG_1, one per direction and combine them to obtain VG_ϵ

Preserves pairwise distances within a factor of $(1+\epsilon)$

– Run the slower FPTAS on $O(\frac{n}{\epsilon} \log n)$ size viability graph VG_ϵ

Primary challenge is to construct the graph VG_1

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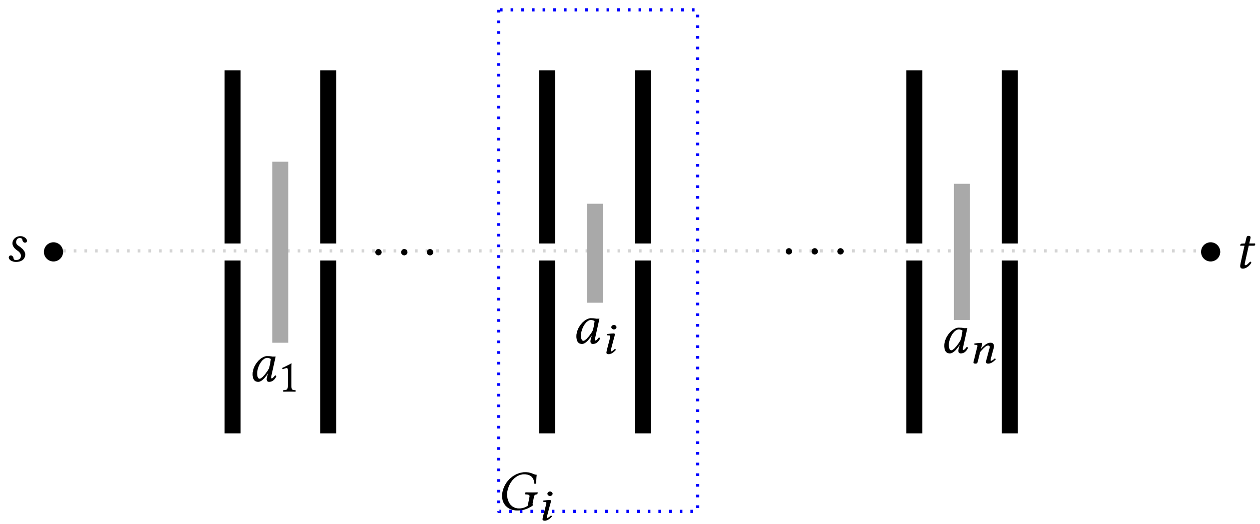
Thanks!

Backup : NP-Hardness

Simple reduction from PARTITION



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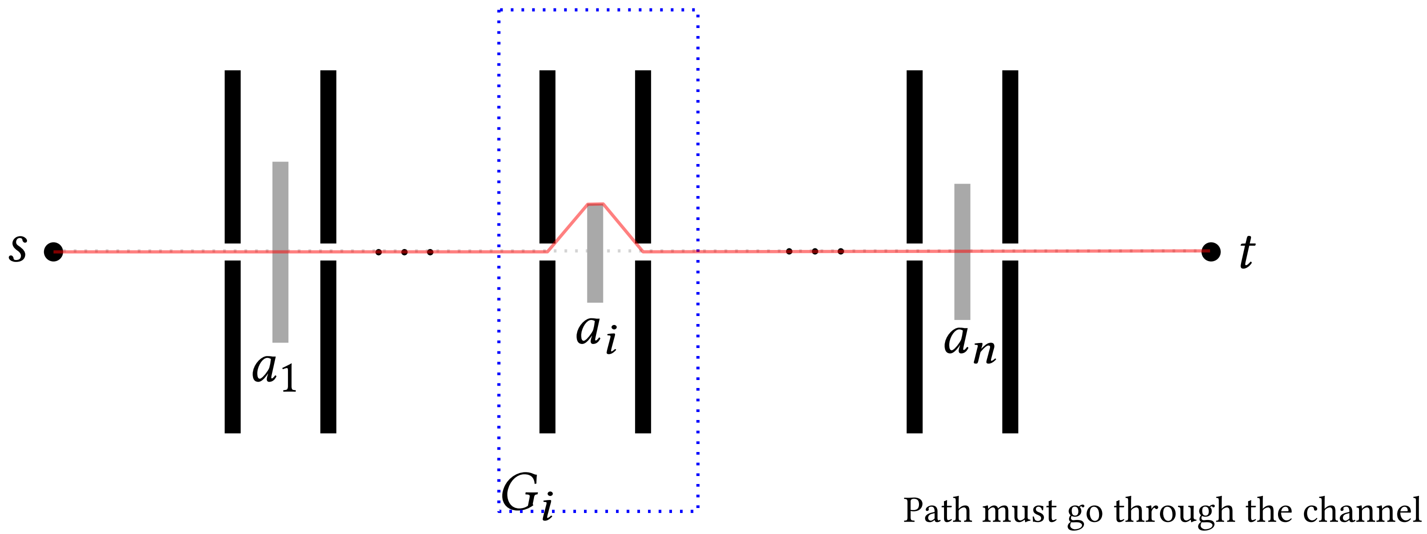

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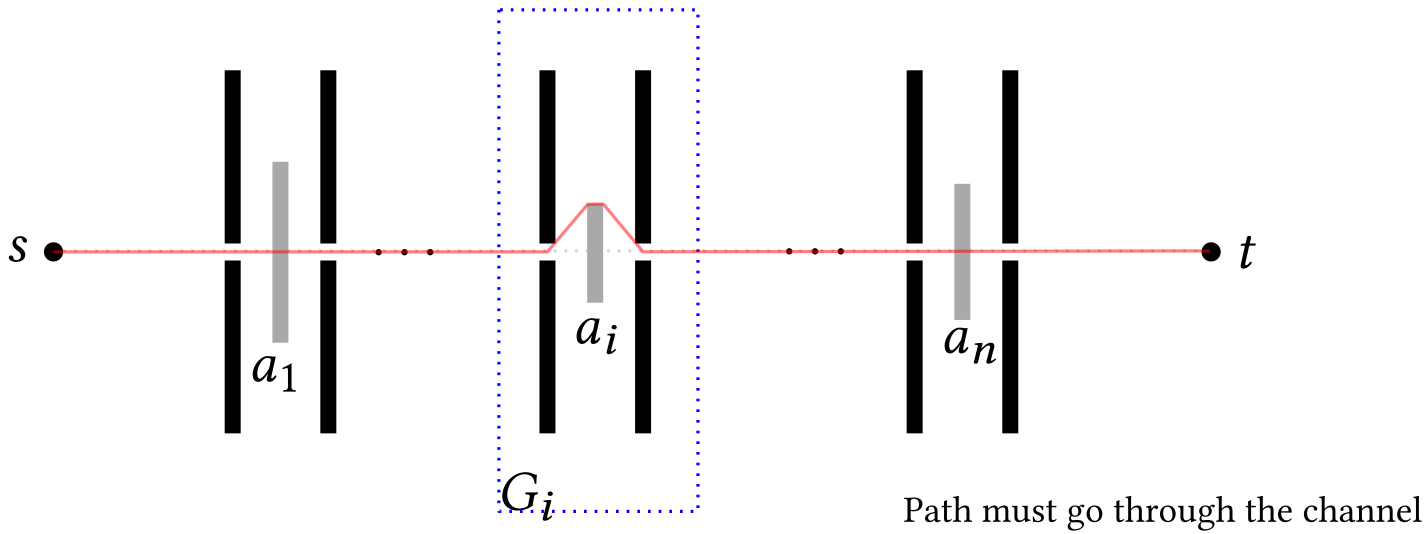

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An s–t path with length at most $L = \frac{1}{2}\sum a_i$ and cost at most $C = \frac{1}{2}\sum a_i$ exists if and only if set A can be partitioned into two equal groups.